

**Problem 1. (15 pts)** Solve the following differential equations and obtain solutions satisfying any given initial conditions.

a.  $2y'' + 3y' + y = 0$ ,  $y(0) = -1, y'(0) = 1$  [  $y = y(t)$  ].

b.  $z'' - 6z' + 10z = 0$ , [  $z = z(t)$  ].

c. Consider the system given by  $\begin{cases} x' = -2x + y \\ y' = -2y \end{cases}$ .

i. Rewrite this as a second order equation.

ii. Solve the second order equation.

| Page         | Pts    | Score |
|--------------|--------|-------|
| 1            | 30 pts |       |
| 2            | 30 pts |       |
| <b>Total</b> | 60 pts |       |

**Problem 2. (15 pts)** Let  $\frac{d}{dt}\mathbf{Y} = \mathbf{A}\mathbf{Y}$ , for  $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$

a. Determine the eigenvalues of  $\mathbf{A}$ .

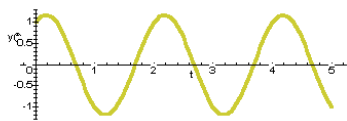
b. Find the eigenvectors of  $\mathbf{A}$ .

c. Find the general solution

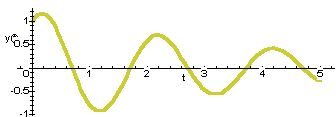
d. Write down the solution to the initial value problem where  $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

**Problem 3. (15 pts)**

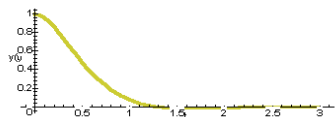
a. Match the solution curves with the below phase portraits, type of eigenvalues, and if the motion is (s) simple harmonic, (u) underdamped, or (o) overdamped.



Roots \_\_\_\_\_  
Phase portrait \_\_\_\_  
Motion \_\_\_\_\_

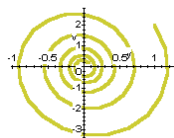


Roots \_\_\_\_\_  
Phase portrait \_\_\_\_  
Motion \_\_\_\_\_

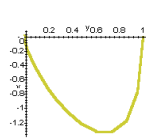


Roots \_\_\_\_\_  
Phase portrait \_\_\_\_  
Motion \_\_\_\_\_

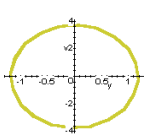
**Phase Portraits**



**Figure A**



**Figure B**



**Figure C**

b. Approximate the period and amplitude for the above periodic case.

c. A 9.0 kg weight hangs from a spring with a spring constant of 81 N/m.

i. What are the natural angular frequency and period of the spring?

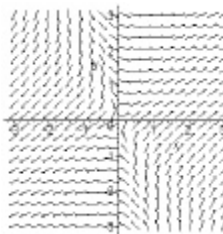
ii. If there is also damping with a damping constant of 10 kg/s, then what is the differential equation governing the motion of the spring?

**Problem 4. (15 pts)**

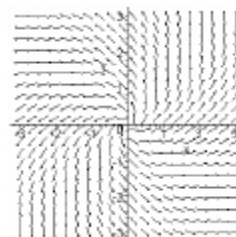
a. Solve:  $y'' + 3y = \sin 3t$ .

b. Describe whatever you can determine from the following direction fields and sketch solution curves.

i)



ii)



iii)

