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Problem 1. Consider the differential equation:

$$\frac{dy}{dx} = x + y,\ y(0) = 1.$$

a. Use Euler's method with $h = 0.2$ to determine the approximate value of *y(1).*

b. Find the exact solution and compare with the answer in part (a).

Problem 2. Solve: $x^2y'' + 5xy' + 4y = 0.$

Page	Points	Score
1	24	
2	26	
3	24	
4	26	
Total	100	

Problem 3. Solve the following first order differential equations:

a. $y' = \sec^2 x,\ y(\pi) = 1.$

b. $xy' + 3y = x.$

c. $xy' = \sqrt{1 - y^2}$.

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Problem 4. Consider the differential equation:

$$y'' + 2y' - 8y = e^{3t},$$

a. Find the solution of the homogeneous equation.

b. Find a particular solution using the method of undetermined coefficients.

c. Find a particular solution using the method of variation of parameters.

d. Solve an initial value problem using the Laplace transform with $y(0) = 0, y'(0) = 1.$

Problem 5.

a. Find the first three nonzero terms of a series solution of the initial value problem:

$$y'' + xy' + 2y = 0,\ y(0) = 0, y'(0) = 1\,.$$

b. Find the first three nonzero terms of a series solution of the differential equation: $xy' + x^2y = 0$

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Problem 6. A tank contains 80 gallons of pure water. A brine solution with 2 lb/gal of salt enters at 2 gal/min, and the well-stirred mixture leaves at the same rate. Find the amount of salt in the tank as a function of time. What is the largest amount of salt left in the tank?

Problem 7. Solve $y'' = 2x(y')^2$, $y(0) = 0$, $y'(0) = -1$.

Problem 8. Let $\frac{dy}{dt} = (c - 2y^2)y$.

a. For $c = 2$ sketch typical solutions in all regions of the ty-plane, and determine the stability of the equilibrium solutions.

b. For $c = -2$ sketch typical solutions in all regions of the ty-plane, and determine the stability of the equilibrium solutions.

c. Draw a bifuraction diagram for this equation.

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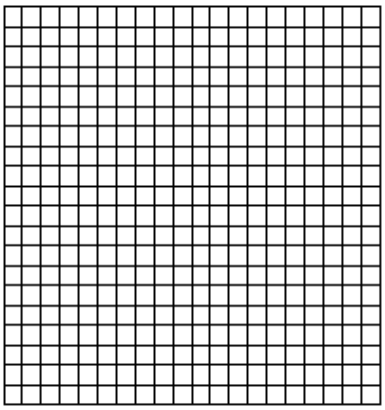
Problem 9. Consider the equation: $y' = x^2 + y$.

a. Describe the nullclines.

b. Describe the isoclines.

c. Sketch the direction field using the above information.



Problem 10. Compute the following: (L = Laplace transform).

a. $L\{\cos 5t + e^{-t}\}$

b. $L^{-1}\{\dfrac{s-1}{s^2-5s+6}\}$

Problem 11. A certain straight line motion is determined by the equation $x'' + 2\gamma x' + 169x = 0$ and the conditions that when $t = 0$, $x = 0$ ft, and $v = 8$ ft/s.

a) Use $\gamma = 12$. Find x in terms of t and sketch the solution curve.

b) Sketch the phase portrait.

c) Use $\gamma = 14$. Find x in terms of t and sketch the solution curve.

d) Find the value of γ that leads to critical damping, determine x in terms of t.