MAT 361 Final Exam

Name ___________________________

Problem 1. Consider the differential equation:

$$\frac{dy}{dx} = xy, \; y(0) = 1.$$

a. Use Euler's method with $h = 0.2$ to determine the approximate value of *y(1).*

b. Find the exact solution and compare with the answer in part (a).

Problem 2. Solve: $x^2 y'' - 3xy' + 4y = 0.$

Page	Points	Score
1	30	
2	30	
3	20	
4	20	
Total	100	

Problem 3. Solve the following first order differential equations:

a. $y' = \frac{xy + y^2}{x^2}, \; y(1) = 1.$

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b. $xy' + 2y = x^2.$

c. $y - xy' + (y')^3 = 0.$

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Problem 4. Solve the differential equation:

$$y'' - y' - 6y = e^t,$$

a. Use the method of undetermined coefficients.

b. Use the Laplace transform with $y(0) = 1, y'(0) = 0$.

c. Use the method of variation of parameters.

Problem 5. Find a series solution of the following equation:

$$y'' + xy' + 2y = 0.$$

Problem 6. Find the indicial equation and interval of convergence of the series solution about the regular singular point of the equation: [Do not solve!]

$$4xy'' + 2(1 + x)y' + y = 0.$$

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Problem 7. Use Newton's Law of cooling to determine the time of death if a corpse is 79°F when discovered at 3:00 P.M. and 68°F three hours later. Assume that the temperature of the surroundings is 60°F. (Normal body temperature is 98.6°F.)

Problem 8. Solve $y'' = 1 + (y')^2$.

Problem 9. Let $\frac{dy}{dt} = (y-a)(y-b)(y-c)$. For each of the cases below, determine the equilibrium solution and sketch possible solutions

(i) $a = b = c = 2$.

(ii) $a = b = 2, c = -1$.

(iii) $a = 2, b = 0, c = -1$.

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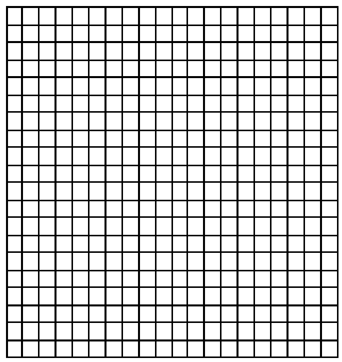
Problem 10. Consider the equation: $\frac{dy}{dx} = x^2 - y^2$.

a. Sketch the slope field.



b. What symmetry does the slope field have?

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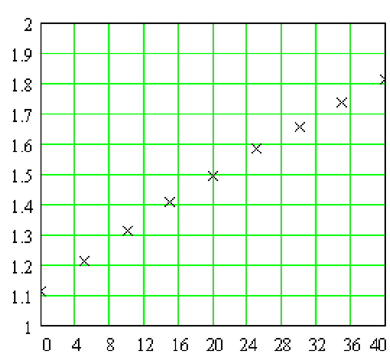
c. When will the solution have a horizontal tangent?

d. Draw isoclines for $m = 1, 4, -4$.

e. Draw the solution curve that passes through (-1,1).

Problem 11. A semilog plot of the World population for five year intervals from 1960 to 2000 is provided below. From this data, determine the population as a function of time. What do you predict for the year 2025? [Note that the data was originally in billions and the ordinate is the natural log of the population. The abscissa is the number of years since 1960.]



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Problem 12. Compute the following: (L = Laplace transform).

a. $L\{\sin 5t + e^{-3t}\}$

b. $L\{te^{2t}\}$

c. $L^{-1}\{\frac{3s+5}{s^2+4}\}$

d. $L^{-1}\{\frac{s-1}{s^2-5s+6}\}$