Laplace Transforms

Laplace Transforms: $Y(s) \equiv L\{y(t)\} \equiv \int_0^\infty y(t)e^{-st}\,dt.$

Properties:

$L\{af(t)+bg(t)\} = aF(s)+bG(s).$

$L\left\{\frac{dy}{dt}\right\} = sY(s)-y(0).$

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s)-sy(0)-y'(0).$

$L\{e^{at}y(t)\} = Y(s-a).$

$L\{H(t-a)y(t-a)\} = e^{-as}Y(s).$

$L\{tf(t)\} = -\frac{d}{ds}F(s).$

Transform Pairs

$f(t)$	$F(s)$	$f(t)$	$F(s)$
c	$\frac{c}{s}$	e^{at}	$\frac{1}{s-a}, s<a.$
t^n	$\frac{n!}{s^{n+1}}, s>0.$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}.$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$t\sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	$t\cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
$H(t-a)$	$\frac{e^{-as}}{s}, s>0$	$\delta(t-a)$	$e^{-as},\ a\ge 0, s>0.$

Convolution

$(f*g)(t) = \int_0^t f(t-u)g(u)\,du.$

$L\{f*g\} = F(s)G(s).$