Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- All short answers and essays should be responded to with full sentences conveying thoughtful responses.
- If you need more space, you may use the **back of a page** and write *On back of page #* in the problem space or the **extra page**. **No other paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient rationale for full credit.

Pay attention to the point distribution. Not all problems have the same weight. Pace yourself!

Page	Pts	Score
1	**31**	
2	**14**	
3	**13**	
4	**10**	
5	**13**	
6	**19**	
Total	**100**	

Bonus: Compute $\phi(6)$. Show the multiplicative group modulo $\phi(6)$? _____.

MAT 346 Exam II **Name** _____

1. (10 pts) Can you name that mathematician?

A B C D E F G H I J

A. _____ E. _____ I. _____

B. _____ F. _____ J. _____

C. _____ G. _____

D. _____ H. _____

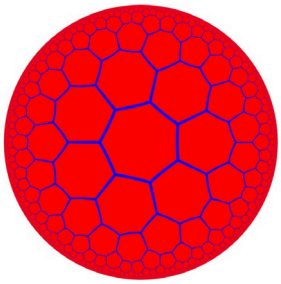
2. (8 pts) Match the Mathematician with the birth country listed below.

Riemann _____ Germain _____ Euler _____ Agnessi _____
Kovalevskaya _____ Lovelace _____ Abel _____ Bolyai _____

a. Norway b. France. c. Russia d. Germany e. England f. Switzerland g. Italy h. Hungary

3. (8 pts) Select the approximate century (1500, 1600, etc.) for each mathematician's birth.
du Châtelet _____ Galois _____ Somerville _____ Newton _____
della Francesca _____ Gauss _____ Fibonacci _____ Agnessi _____

4. (2 pt) What is the Schläffli tiling number for the below tiling? _____



5. (3 pts) What are the cube roots of i?

6. (5 pts) Answer the following questions by filling in the blank.

   a. Simplify $e^{-\pi i/3}$ to real numbers or radicals. _____

   b. Name $f(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\cdots.$ _____

   c. What is $\int_0^x \frac{dt}{\sqrt{(t-a)(t-b)(t-c)}}$ called? _____

   d. What is the geometry of the disk in Problem 4? _____

   e. What does $\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}++\frac{1}{n}-\ln n\right)$ give? _____

7. (6 pts) Consider the following related series with $|x|<1$:

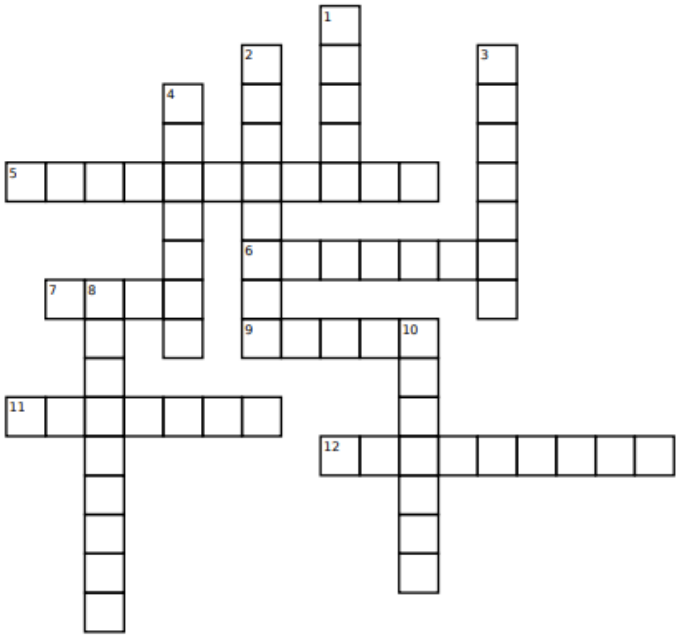
   a. Sum the series: $1-x+x^2-x^3+\cdots+(-x)^k+\cdots.$

   b. Use your answer to find the series representation of $\ln(1+x)$.

   c. Explain how Newton used this to find the series expansion for e^x.

8. (3 pts) Given $\int_0^1 \frac{dt}{\sqrt{1-t^4}}=\frac{\pi}{2\cdot AGM\left(1,\sqrt{2}\right)}$, find the area of the lemniscate $r^2=\cos 2\theta$ to five decimal places. [Hint: First find $AGM\left(1,\sqrt{2}\right)$.]

9. (6 pts) Fill in the names of the mathematicians.



Down:

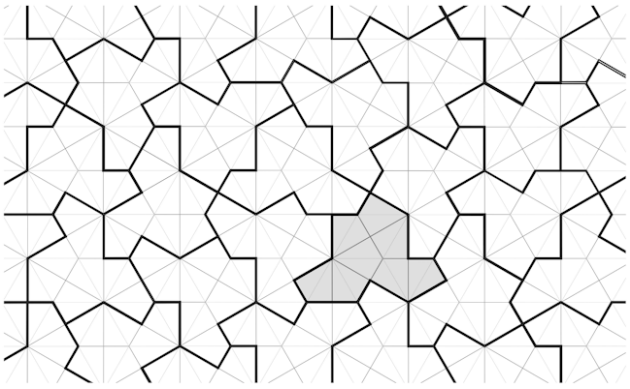
1. He wrote Disquisitiones arithmeticae.
2. Wrote first computer program.
3. Saved Gauss from the French.
4. The father of accounting.
8. A mathematical dynasty from Basel.
10. Gave us the most significant open math problem.

Across:

5. His praise was sung by Tom Leher.
6. First woman to write a mathematics textbook.
7. His groups commute.
9. Solved the Basel problem.
11. Black holes and tiling the plane.
12. He understood the significance of 1729.

10. (4 pts) The below tiling was published in March 2023.
    a. Name the gray tile. _____
    b. Why was this important in 2023?

    c. Are these tiles all the same? If not, which ones are not?



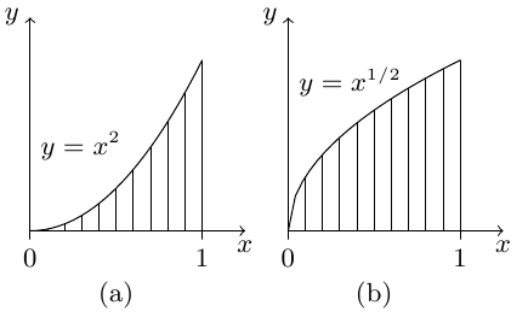
11. (3 pts) For a triangle on a surface, the defect is $D = \alpha + \beta + \gamma - \pi$.
    a. For what surface is it positive? _____
    b. For what surface is it negative? _____
    c. What is D proportional to? _____

12. (5 pts) Employing Cardano's method of solving the depressed cubic $x^3 - 87x + 284 = 0$, one obtains $x = \sqrt[3]{-142 + 65i} + \sqrt[3]{-142 - 65i}$. However, the roots are $x = 4, -2 \pm 5\sqrt{3}$.

    a. Show that $(2 + 5i)^3 = -142 - 65i$.

    b. What does this tell you about Cardano's solution?

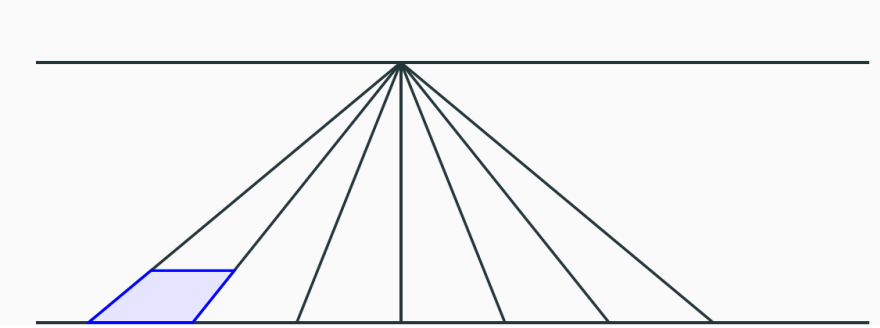
13. (5 pts) Use the figures to answer the following questions.



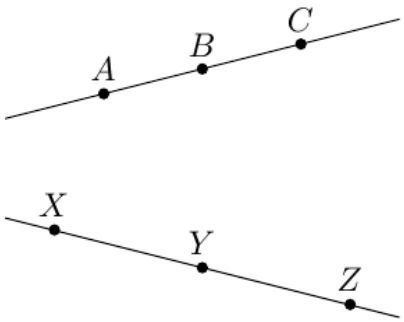
    a. Mathematicians knew how to compute $\int_0^1 x^2\,dx$. Use Figure (a) to demonstrate how this was done. [Hint: $\sum_{k=1}^{N} k^2 = \frac{1}{6}N(N+1)(2N+1)$.]

    b. In *Arithmetica Infinitorium* John Wallis obtained equivalent results, but also treated fractional powers. Use Figure (b) to describe how he computed $\int_0^1 x^{1/2}\,dx$.

14. (4 pts) Consider the below example of perspective.
    a. What is the top line called? _____
    b. What is the point to which the other lines converge? _____
    c. You want to tile the "road" below. Explain how one does this so that the tile perimeters are drawn with correct perspective.



15. (4 pts) Consider two sets of collinear points as shown below.
    a. What does Pappus' Theorem say about the lines connecting the points A, B ,C with X, Y, and Z?
    b. Demonstrate this.
    c. What did Pascal contribute to the problem?



16. (2 pts) Consider the elliptic curve $y^2 = x^3 + x + 1$. Write this in homogeneous coordinates, (X, Y, Z).

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17. (3 pts) What was Torricelli's trumpet and why did it upset people like Thomas Hobbes?

18. (4 pts) What are the following:

    a. The Fundamental Theorem of Algebra.

    b. Descartes' Factor Theorem.

19. (2 pts) Explain how Johann Bernoulli related the tangent function to the logarithm?

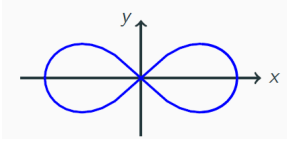
20. (4 pts) Biographical questions:
    a. Who was Lord Byron's daughter? _____
    b. Who fought a duel and lost? _____
    c. Who found 61 pairs of amicable numbers? _____
    d. In what country were the first doctorates obtained by women? _____

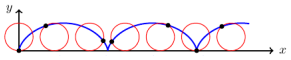
21. (6 pts) Match the topic of study with the mathematician.

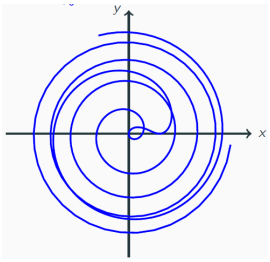
A. Propagation of fire. B. Elasticity. C. 4D polytopes.
D. Spinning tops. E. Geography text. F. Mortality rates.

Emile Du Chatelet ____	Sophi Germain ____	Sonia Kovalevskya ____
Alicia Boole Stott ____	Florence Nightingale ____	Somerville ____

22. (3 pts) Explicitly name the blue curves:







_____ _____ _____

Extra Page