Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- All short answers and essays should be responded to with full sentences conveying thoughtful responses.
- If you need more space, you may use the **back of a page** and write *On back of page #* in the problem space or the **extra page**. **No other paper is allowed.**

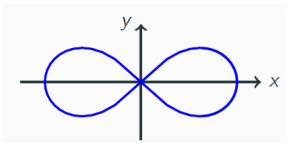
Try to answer as many problems as possible. Provide as much information as possible. Show sufficient rationale for full credit.

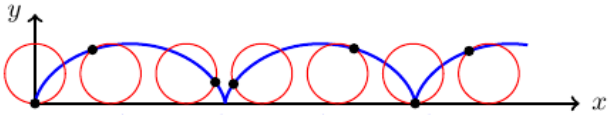
Pay attention to the point distribution. Not all problems have the same weight. Pace yourself!

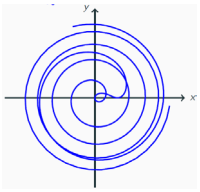
Page	Pts	Score
1	**31**	
2	**14**	
3	**12**	
4	**19**	
5	**16**	
6	**17**	
Total	**109**	

Note: $\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum_{k=1}^{n} k^3 = \frac{1}{4}n^2(n+1)^2$.

Bonus: Name the curves:







_____ _____ _____

MAT 346 Exam II **Name** _____

1. (8 pts) Match the Mathematician with the country listed below.

   Lovelace _____ Leibniz _____ Euler _____ Cauchy _____
   Kovalevskaya _____ Riemann _____ Abel _____ Agnesi _____

   a. Norway b. France. c. Russia d. Germany e. England f. Switzerland g. Italy.

2. (8 pts) Select the approximate century (1500, 1600, etc.) for each mathematician.
   Germain _____ Galois _____ Bombelli _____ Newton _____
   Descartes _____ Lobachevsky _____ Gauss _____ Brunelleschi _____

3. (10 pts) Can you name that mathematician? [Hint: The names have been used above.]

   A B C D E F G H I J

   A. _____ E. _____ I. _____

   B. _____ F. _____ J. _____

   C. _____ G. _____

   D. _____ H. _____

4. (5 pts) Answer the following questions by filling in the blank.

   a. Simplify $e^{i\pi/6}$. _____

   b. What is $\int_0^x \frac{dt}{\sqrt{(t-a)(t-b)(t-c)}}$ called? _____

   c. Give a cube root of unity not equal to one. _____

   d. Give a Germain Prime. _____

   e. What is Fermat's Last Theorem? _____

5. (6 pts) Consider the following family of series: $f(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\cdots.$

   a. What is the name of the series $f(1)$? _____

   b. What is the value of $f(1)$? _____

   c. The series $f(2)$ was the subject of a famous problem.

      i. What was the problem called? _____

      ii. Who solved it? _____

      iii. What is the value of $f(2)$? _____

   d. The general function, $f(s)$, is called the _____

6. (6 pts) When employing Cardano's method to solve the cubic $x^3+x=2,$ one obtains $x=\sqrt[3]{1+\frac{2}{3}\sqrt{\frac{7}{3}}}+\sqrt[3]{1-\frac{2}{3}\sqrt{\frac{7}{3}}}.$ However, one can show that one of the roots is $x=1.$

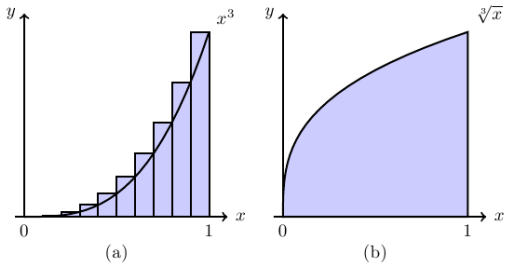
   a. Verify that this is a root.

   b. Find the other two complex roots.

   c. Knowing that $\left(\frac{1}{2}\pm\frac{1}{2}\sqrt{\frac{7}{3}}\right)^3=1\pm\frac{2}{3}\sqrt{\frac{7}{3}},$ how does the Cardano solution simplify?

7. (2 pts) Show how the projective line is topologically a circle.

8. (5 pts) Use the figures to answer the following questions.



   a. Wallis knew how to compute $\int_0^1 x^3\,dx$. Use Figure (a) to demonstrate how this might have been done.

   b. In *Arithmetica Infinitorium* Wallis also treated fractional powers. Use Figure (b) to describe how he computed $\int_0^1 x^{1/3}\,dx$.

9. (7 pts) Newton used power series to find series expansions for functions and their inverses.

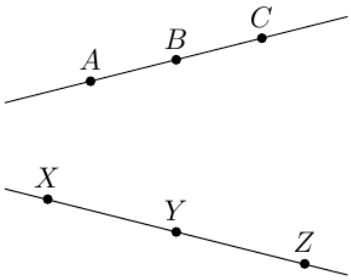
   a. Sum the geometric series: $1-t^2+t^4-\cdots+\left(-t^2\right)^n+\cdots, |t|<1.$

   b. Use the integral $\int_0^x \frac{dt}{1+t^2}$ to obtain the Maclaurin series for $\tan^{-1} x$.

   c. How is this used to obtain a decimal expansion for π?

   d. Who proved π was irrational? _____

10. (3 pts) Construct lines connecting the points A, B ,C with X, Y, and Z and demonstrate Pappus' Theorem.



11. (9 pts) There are three types of geometry. Fill in the table.

A B C

	A	B	C
Name the geometry.			
Who's name is attached?			
What is the sum of the angles of a triangle in this geometry			

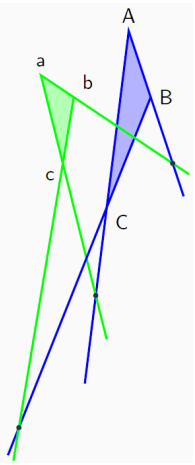
12. (4 pts) Consider the elliptic curve $y^2 = x^3 + x + 1$.
    a. Write this in the homogeneous coordinates *X, Y,* and *Z*.

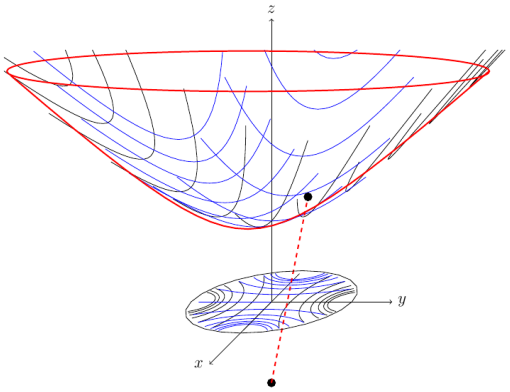
    b. Find the "point at infinity."

13. (3 pts) Find $AGM(1,5)$ to three decimal places.

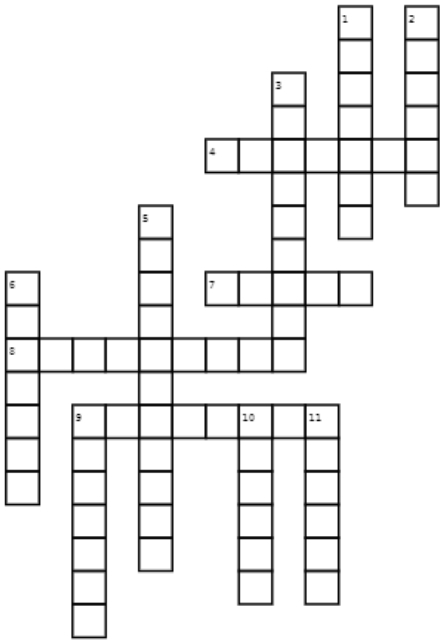
14. (4 pts) Use the figures below to answer the questions.
    a. Desargues Theorem uses the below triangles. What does it conclude?

    b. In the right figure, what type of surface is shown? _____
    c. What is the disk below the surface called? _____





15. (12 pts)

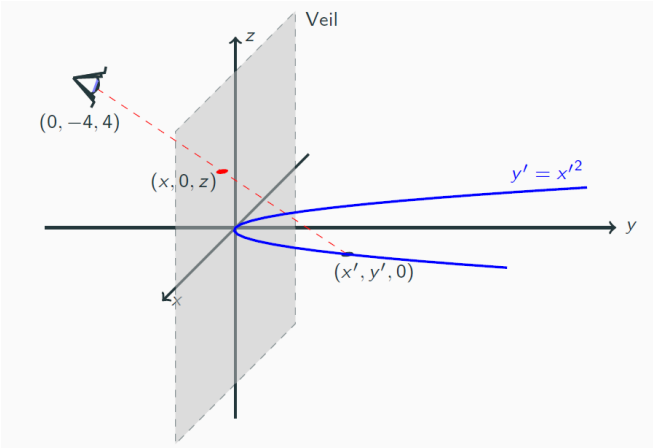


Down:
1. Saved Gauss' life.
2. Died in a duel.
3. Euler's teacher.
5. Played with tops.
6. Father of accounting.
9. Gave us the integral sign.
10. First woman math professor.
11. Dutch graphic artist.

Across:
4. Solved the quartic equation.
7. Solved the Basel Problem.
8. Promoted infinitesimals.
9. First programmer.

16. (4 pts) Consider Alberti's Veil depicted below.
    a. Find the image $(x,0,z)$ in the Veil of the point $x'=2, y'=4$.

    b. Sketch the image of the parabola in the Veil.



17. (10 pts) Name an important contribution to mathematics by these people:
    a. Cavalieri _____
    b. Galois _____
    c. Nightingale _____
    d. Lobachevsky _____
    e. Descartes _____
    f. Cauchy _____
    g. Kovalevskaya _____
    h. Brunelleschi _____
    i. Lovelace _____
    j. Daniel Bernoulli _____

18. (3 pts) Name three famous mathematicians from the 1800s who passed away by 40.