

Introduction to the History of Mathematics

Fall 2022 - R. L. Herman



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Early Civilizations - Babylonian, Egyptian, Chinese, Indian, Islamic

Renaissance Mathematics - 15th-16th Centuries

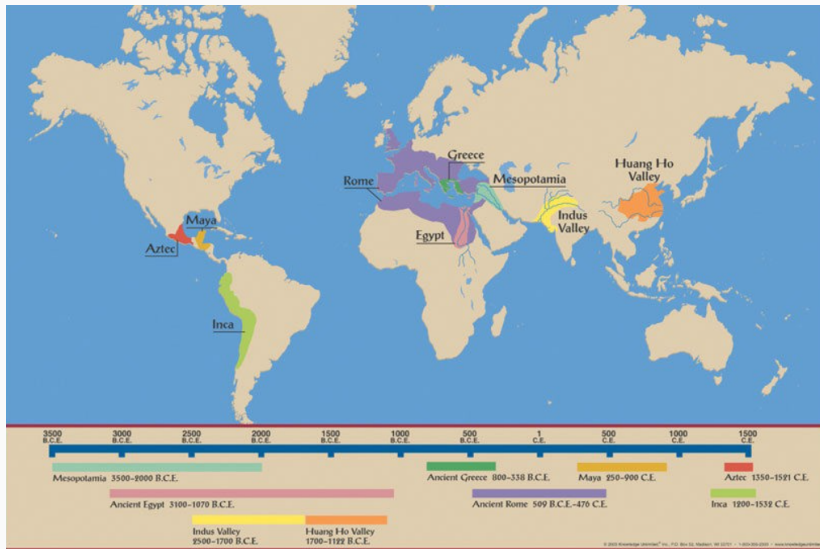
The Rise of Calculus - 17th Century

Exploiting Calculus - 18th Century

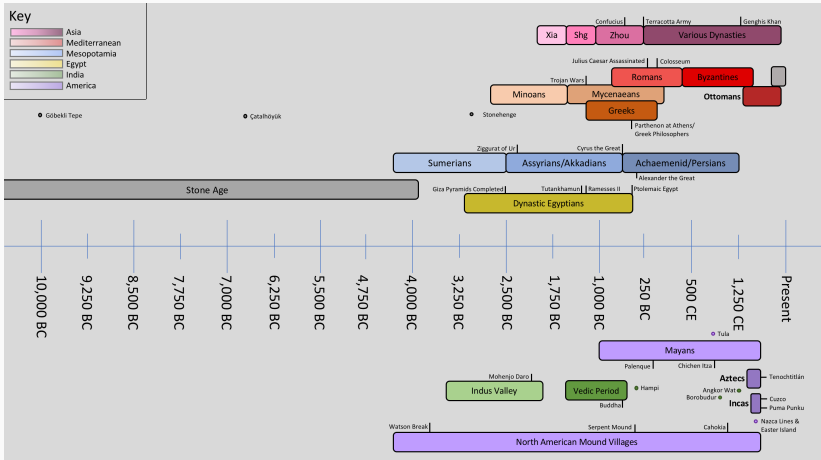
The Birth of Rigor - 19th Century

The Modern Era - 20th Century

Civilizations - How Did Mathematics Develop?



A Civilization Timeline



Some Early Civilizations

- Egypt (3150-30 BCE)
- Mesopotamia (3100-539 BCE)
- Chinese (1766 BCE-220 CE)
- Indian Mathematics (500-1200)
- Mayan Mathematics (250-900)
- Aztecs and Incans (1345-1560)

Arithmetic, Geometry, No proofs.
Problems were practical or recreational.

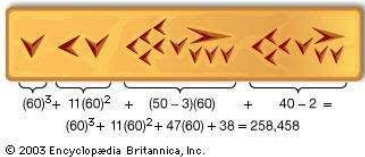


Figure 1: Babylonian Math - Base 60

Greek Civilization

- Deductive Reasoning
 - Definitions, Axioms
 - Propositions via logic
- Geometry, Trigonometry, Astronomy, Numbers, Conics
- Thales (624-546 BCE)
- Pythagoras (6th Century BCE)
- Euclid (4th Century BCE)
Elements - geometry, numbers
- Archimedes (3rd Century BCE)
- Apollonius (2nd Century BCE)
- Heron (10-70), Diophantus (200-284), Pappas (290-350), Hypatia (370-415)



Figure 2: Euclid

Chinese and Indian

- Chinese Mathematics
1300 BCE - 1800 CE
 - Pythagorean Thm
 - π estimates
 - Volumes, Applications
 - Pascal's Triangle
 - Chinese Remainder Thm
- Indian Mathematics 1200 BCE,
mostly 500-1200 CE
 - Geometry
 - Trigonometry
 - Power series
 - Astronomy
 - π estimates
 - Number system, 0
 - Pell's Equation



Figure 3: Liu-Hong

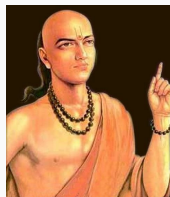


Figure 4: Aryabhata

Middle Eastern Mathematics - 700-1200 CE

European Dark Ages - 400-1200 CE

- Founding of Islam - 7th Century
- Islamic mathematicians preserve/translate Greek/Asian mathematics into Arabic
- Arabic Numerals by 1000 CE
- Persian mathematicians
 - al-Khwarizmi (780-850)
 - Algebra (al-Jabr)
 - Omar Khayyam (1048-1131)
 - geometric solution of cubic

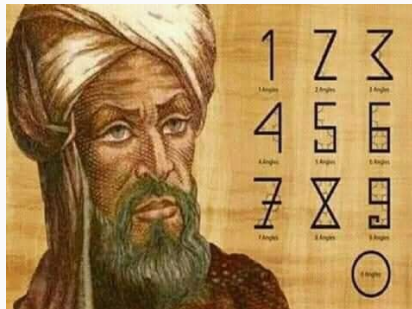


Figure 5: al-Khwarizmi

Around 10th Century - Middle Eastern Mathematics brought to Spain.

It takes 300 years to accept Hindu-Arabic numerals. - Fibonacci - 1202

The Renaissance

The Renaissance 1400-1600



Leonardo of Pisa (1175-1250)

Black Death (1347-1351)



Gutenberg's Press

Printing Press (1440)

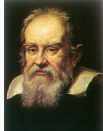
Luca Pacioli (1445-1517)



Giovanni di Medici
Banking (1397)

Leonardo da Vinci (1452-1519)

Galileo (1564-1642)



Perspective in Art (1377-1492)



Bruneschelli, Alberti, Piero della Francesca



Reformation (1517)

Cardano (1501-1576)



Tartaglia (1500-1557)



Michelangelo (1475-1564)

Beyond Numerals

- Fractions 4000 years ago
- Sexagesimal (base 60) into 17th century
- Decimal (base 10)
 - al-Uqlidisi - (920-980)
 - al-Kashi (1380-1429)
 - Simon Stevin (1548-1620)
- Logarithms
 - John Napier (1550-1617)
 - used a stange base
 - Henry Briggs (1561-1630)
 - Base 10 Tables
 - 54 square roots of 10 (30 decimal places)
 - Tables - 14 decimal places

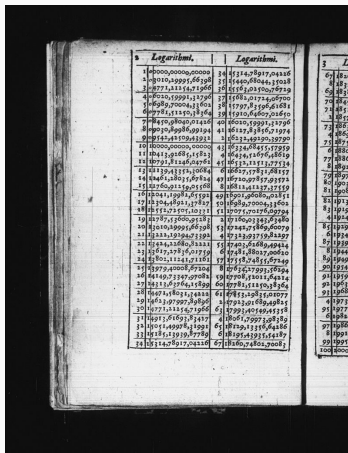


Figure 6: Briggs's Tables

- Fibonacci (Leonardo of Pisa)
(1170-1250) *Liber Abaci*
- Equation Solving contests
- Solutions of cubic and quartic
 - Depressed cubic
del Ferro (1465-1526)
 - Cubic and quartic equations
Tartaglia (1500-1557)
Cardano (1501-1576)
Ars Magna
Ferrari (1522-1565)
- Bombelli (1526-1572)
 - Complex numbers
- Viète (1540-1603)
Adriaan van Roomen Problem



Figure 7: Cardano and Tartaglia
Fight of the Century!

Unification of Geometry and Algebra

- Symbolic Algebra
 - Rhetorical until 15th century
 - Syncopated/abbrev. - 1500
 - Symbolic algebra developed 16-17th century
- Unification
 - Oresme (1320-1382) -
Velocity-time graphs, $\sum \frac{1}{n}$
 - Descartes (1596-1650)
 - Rep. curves by equations
 - Coordinate systems -
published *The Method*
 - Made use of variables which
can vary continuously - lines.
 - Fermat (1607-1665)
 - Rep. equations by curves



Figure 8: Fermat and Descartes

The Rise of Calculus

- Archimedes - 3rd century BCE
- Kepler (1571-1630)
- Cavalieri (1598-1647)
- Fermat (1607-1665)
- Wallis (1616-1673)
- Pascal (1623-1662)
- Barrow (1630-1677)
- Wren (1632-1723)
- Gregory (1638-1675)
- Newton (1642-1726)
 - *Principia* 1687
- Leibniz (1646-1716)
 - Notation $\frac{d}{dx}$, \int

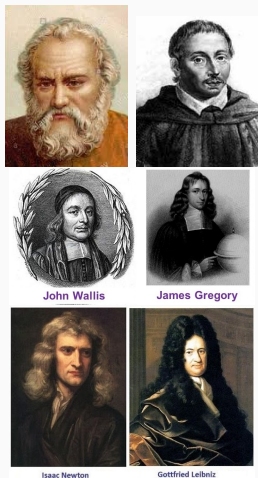


Figure 9: Archimedes, Cavalieri, Wallis, Gregory, Newton, and Leibniz

The Infinitesimal

- Hippasus 500 BCE
 - Pythagorean
 - $\sqrt{2}$ irrational
- Introduction of Infinitesimals
 - Cavalieri and Torricelli
 - Stevin, Wallis, Harriot
- Critics
 - Jesuits in Italy
 - George Berkeley (1685-1753)
The Analyst, - A Discourse Addressed to an Infidel Mathematician, 1734
 - “Infinitesimals undermine mathematics and rationality”
- Augustin-Louis Cauchy - 1821

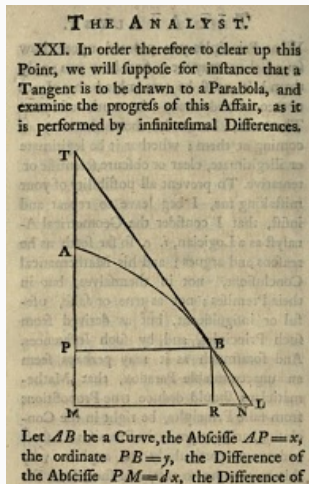


Figure 10: Berkeley's *The Analyst*

Exploiting Calculus

- Bernoulli Family
- Euler (1707-1783)
- Laplace (1749-1827)
- Neptune discovered using math - 1846, Le Verrier (1811-1877)

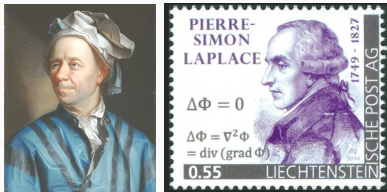


Figure 11: Euler and Laplace

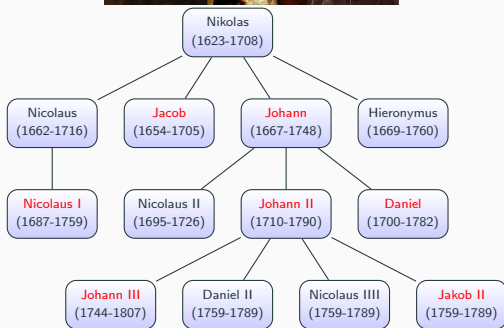
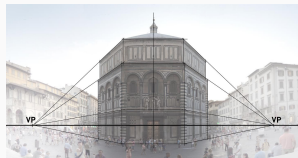


Figure 12: The Bernoulli Family

Evolution of Geometry: Projective Geometry and Topology

- Perspective in Art
 - Bruneschelli (1377-1446)
 - Leon Alberti (1404-1472)
 - Girard Desarges (1591-1661)
- Birth of Topology
 - Euler - Königsberg bridge
Geometry without distance
 - Euler Characteristic
 $\chi = V - E + F$, and
classification of surfaces
 $\chi = 2 - 2g$, genus
- Birth of Knot Theory
 - Gauss - Intertwining curves
 - Scottish physics and Knots -
Tait's smoke rings.



The Birth of Rigor - 19th Century

Non-Euclidean Geometry

- Parallel Postulate
- Hyperbolic Geometry
 - Nikolai Lobachevsky (1792-1856)
 - Johann Bolyai (1802-1860)
 - Johann Carl Friedrich Gauss (1777-1855)
- Elliptic Geometry
 - Georg Friedrich Bernhard Riemann (1826-1866)
 - Prince of Mathematicians
- By 1870's Euclid in doubt!



Figure 13: Gauss, Lobachevsky, Bolyai

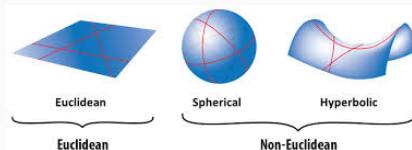


Figure 14: Different Geometries

19th Century Group Theory

The search for the general solution of the quintic.

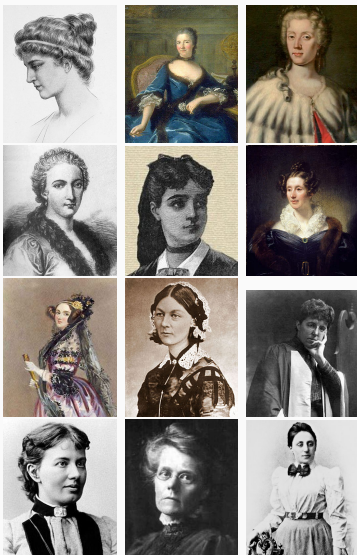
- Joseph-Louis Lagrange (1736-1813)
- Johann Carl Friedrich Gauss (1777-1855)
- Paolo Ruffini (1765-1822) - proof of unsolvability
- Augustin Cauchy (1789-1857)
- Niels Henrik Abel (1802-1829)
- Évariste Galois (1811-1832)
- Arthur Cayley (1821-1895)
- Camille Jordan (1838-1922)



Figure 15: Abel and Galois

Famous Women Mathematicians Before 1900

- Hypatia of Alexandria (c. 350-415)
- Émilie du Châtelet (1706-1749)
- Laura Bassi (1711-1788)
- Maria Agnesi (1718-1799)
- Sophie Germain (1776-1831)
- Mary Fairfax Somerville (1780-1872)
- Ada Lovelace (1815-1852)
- Florence Nightingale (1820-1910)
- Charlotte Angas Scott (1848-1931)
- Sofia Kovalevskaya (1850-1891)
- Alicia Boole Stott (1860-1940)
- Amalie 'Emmy' Noether (1882-1935)



19th Century Analysis and Set Theory

- Jean-Baptiste Joseph Fourier (1768-1830)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Karl Weierstrass (1815-1897)
- George Boole (1815-1864)
- Georg Friedrich Bernhard Riemann (1826-1866)
- Richard Dedekind (1831-1916)
- Georg Ferdinand Ludwig Philipp Cantor (1845-1918)
 - Founder of set theory
 - Defined infinite sets



Figure 16: Gauss and Riemann

19th Century Number Theory

- Marie-Sophie Germain (1776-1831)
- Johann Carl Friedrich Gauss (1777-1855)
Disquisitiones Arithmeticae - 1801
- Adrien-Marie Legendre (1752-1833) and Peter Gustav Lejeune Dirichlet (1805-1859) prove Fermat's Last Theorem for $n = 5$ in 1825
 - Dirichlet, $n = 14$ in 1832.
- Riemann Hypothesis, distribution of primes - 1832.
- Charles Jean de la Vallée-Poussin and Jacques Hadamard - Prime Number Theorem. 1896
- H. Minkowski: Geometry of Numbers, 1896.



Figure 17: Sophie Germain, Adrien-Marie Legendre

The Modern Era

We stop at the turn of the 20th Century: the evolution of mathematics, set theory, physics revolutions, Bourbaki, Hilbert's 23 Problems.

Explore mathematics prizes: Fields Medal, Abel Prize, Wolf Prize, Millenium Prize.

Other Sites

- Chronology of 20th Century Mathematicians
- Greatest Mathematicians born between 1860 and 1975
- Pictures of Famous 20th Century Mathematicians
- The Story of Math Website



Figure 18: Hilbert, Gödel, Uhlenbeck, Ramanujan, Wiles, Mirzakhani, Shannon, Russell, Noether