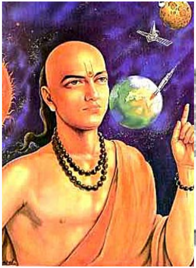
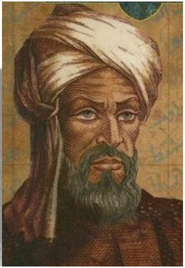
Early Asian Mathematics

Fall 2022 - R. L. Herman

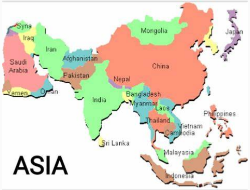




Overview

China

- Unique development
- *Zhoubi Suanjing* - c. 300 BCE
- *Tsinghua Bamboo Slips*, - decimal times table. 305 BCE
- Chinese abacus (<190 CE)
- After book burning (212 BCE), Han dynasty (202 BCE–220) produced mathematics works.



India

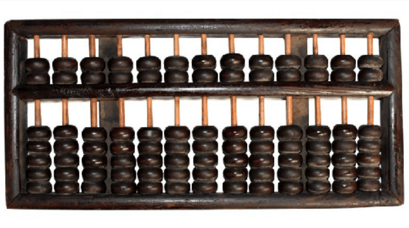
- *Pingala* (3rd–1st cent. BCE) - binary numeral system, binomial theorem, Fibonacci numbers.
- *Siddhantas*, 4th-5th cent. astronomical treatises, trigonometry.

Arabian-Islamic (330-1450)

- Preserved Greek texts
- 7th-14th cent. Development of algebra, etc.
- Hindu-Arabic numerals

Bamboo Slips and Suanpan

- *Tsinghua Bamboo Slips*, - Decimal Times Table. 305 BCE. By People from Warring States Period (476-221 BC), On right - Public Domain Image
- *Suanpan*, Chinese abacus (<190 CE), Below.



Chinese Dynasties

- Xia Dynasty (c. 2070-1600 BCE)
- Shang Dynasty (c. 1600-1046 BCE)
- Zhou Dynasty (c. 1046-256 BCE)
  Periods: Western Zhou, Spring and Autumn (770), Warring States (475)
- Qin Dynasty (221-206 BCE)
- Han Dynasty (206 BCE-220 AD)
- Six Dynasties Period (220-589)
- Sui Dynasty (581-618)
- Tang Dynasty (618-907)
- Five Dynasties and Ten Kingdoms (907-960)
- Song Dynasty (960-1279)
- Yuan Dynasty (1279-1368)
- Ming Dynasty (1368-1644)
- Qing Dynasty (1644-1912)

Figure 1: Liu Hui's circle.

Before Qin Dynasty (< 221 BCE)

- Known from myths and legends.
- Grasped numbers, figures.
- Quipu knots to record events, numbers.
- Used gnomon and compass.
- Artifacts - patterns on bone utensils.
- Plastrons (turtle shells) and oracles bones found in 1800's from Shang dynasty.
- Earliest known Chinese writing.
- Numbers up to 30,000. Special characters for numbers like 30,000, 20,000, 10,000, etc. [No zero.]
- Bone script from Zhou dynasty.

Calculations

- Ancient tools.
  - Babylonians - clay tablets.
  - Egyptians - hieroglyphs, papyri.
  - Indian, Arab - sand boards.
  - Chinese - counting rods.
- Short bamboo rods.
  - Han Dynasty: 1/10" × 6".
  - Sui Dynasty: 1/5" × 3".
  - No later than Warring States period.
  - Arranged - decimal place-value.
- Arithmetic operations. $+ - \times \div$
- Nine-nines rhyme - seen on bamboo strips. Spring and Autumn period (770-476).

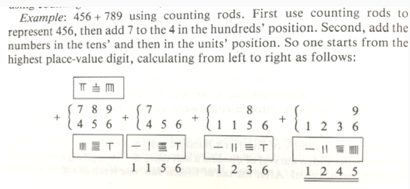
Chinese Numbers

rod	char.	digit	name
𝍩	一	1	i
𝍪	二	2	erh
𝍫	三	3	san
𝍬	四	4	ssu
𝍭	五	5	wu
𝍮	六	6	liu
𝍯	七	7	ch'i
𝍰	八	8	pa
𝍱	九	9	chiu
|	十	10	shih
—	百	100	pai
	千	1000	ch'ien

Arithmetic Operations

Clips from *Chinese Mathematics A Concise History* by Yan and Shiran.

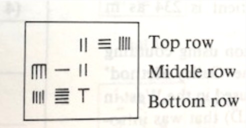
Addition - $456 + 789$



Example: 456 + 789 using counting rods. First use counting rods to represent 456, then add 7 to the 4 in the hundreds' position. Second, add the numbers in the tens' and then in the units' position. So one starts from the highest place-value digit, calculating from left to right as follows:

Multiplication - 234×456

Start with 2×456



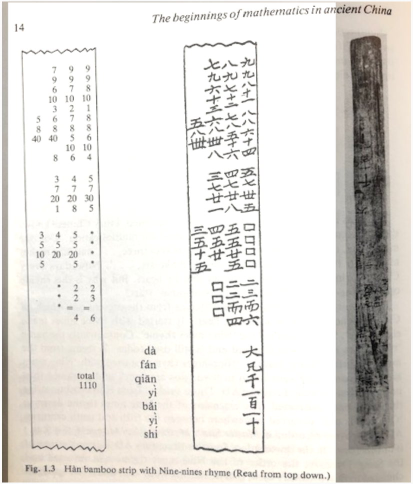
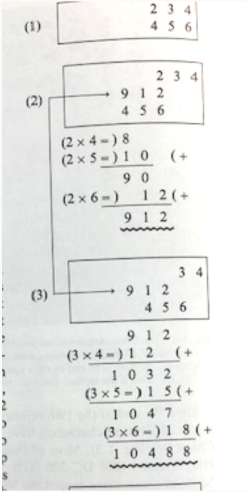


Figure 2: Nine-nines Rhyme.

Multiplication 234×456

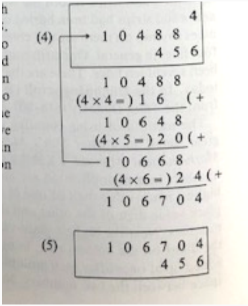

(1)
2 3 4
4 5 6

(2)
2 3 4
9 1 2
4 5 6

(2 × 4 =) 8
(2 × 5 =) 1 0 (+
9 0
(2 × 6 =) 1 2 (+
9 1 2

(3)
3 4
9 1 2
4 5 6

9 1 2
(3 × 4 =) 1 2 (+
1 0 3 2
(3 × 5 =) 1 5 (+
1 0 4 7
(3 × 6 =) 1 8 (+
1 0 4 8 8


(4)
4
1 0 4 8 8
4 5 6

1 0 4 8 8
(4 × 4 =) 1 6 (+
1 0 6 4 8
(4 × 5 =) 2 0 (+
1 0 6 6 8
(4 × 6 =) 2 4 (+
1 0 6 7 0 4

(5)
1 0 6 7 0 4
4 5 6

The Answer: $234 \times 456 = 106704$.

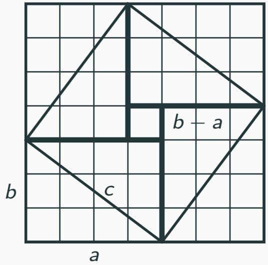
Early Mathematics

- Early works - construction techniques and statistics
- Fractions, measurements, angles, geometry, limit.
- Education - Six Gentlemanly Arts of the Zhou Dynasty (*Zhou Li - The Zhou Rites*).
  - Ritual,
  - Music,
  - Archery,
  - Horsemanship,
  - Calligraphy,
  - Mathematics.

Increased productivity in Han Dynasty (206 BCE-220 CE).

Zhoubi suanjing

- *Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*,
- Written 100 BCE-100 CE.
- Uses **Gōugǔ Theorem**.
  [Around 1100 BC, Western Zhou period; Shanggao first described the Theorem.]
- Shadow gauges - vertical stakes for observing sun's shadow in Zhou.
- Fractions
  - 7 extra lunar months in 19 yr.
  - So, $12\frac{7}{19}$ mo/yr gives $29\frac{499}{940}$ da/mo.
  - Moon goes $13\frac{7}{19}^{o}$ per day. Then, 1 year $= 365\frac{1}{4}$ days.



$$c^2 = (b-a)^2 + 4(\tfrac{1}{2}ab) = a^2 + b^2$$

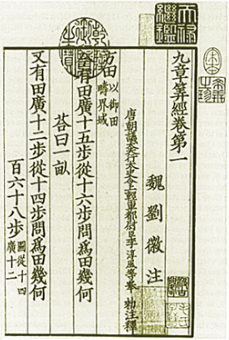
- Gǔ = vertical gauge.
- Gōu = shadow.
- Xián = Hypotenuse.

Jiuzhang Suanshu *The Nine Chapters on the Mathematical Art.*

- Complete by Eastern Han Dynasty (AD 25-220),
- Composed by generations of scholars starting 10th century BCE.
- Designated by Imperial Court as nation's standard math text - During Tang (618-907) and Song (960-1279) Dynasties.
- 246 word problems in 9 chapters on agriculture, business, geometry, engineering, surveying.
- Proof for the Pythagorean theorem.
- Formula for Gaussian elimination.
- Provides values of π. [They had approximated as 3.]
- Used negative numbers.

Chapters of the Mathematical Art

1. Rectangular Fields (Fangtian);
2. Millet and Rice (Sumi);
3. Proportional Distribution (Cuifen);
4. The Lesser Breadth (Shaoguang);
5. Consultations on Works (Shanggong);
6. Equitable Taxation (Junshu);
7. Excess and Deficit (Yingbuzu);
8. The Rectangular Array (Fangcheng); and
9. Base and Altitude (Gougu)



Chapters 1 and 9 summed up the accumulated knowledge of geometry and introduced a proposition - the Pythagorean theorem.

Influenced mathematical thought in China for centuries, introduced into Korea during the Sui Dynasty (581-618) and into Japan during the Tang Dynasty.

Notes and Commentaries

- Liu Hui (c. 225-295), mathematician and Li Chunfeng (602-670), astronomer and mathematician, known for commentaries on *The Nine Chapters on the Mathematical Art*.
- Chapters 2, 3 and 6: Proportion problems. 1st time.
- Chapter 7: The rule of False Double Position for linear problems. 1st time. [In Europe in the 13th century, Fibonacci.]
- World's earliest systematic explanation of fractional arithmetic.
- Chapter 8: Gaussian elimination.
  $> 1,500$ years before Carl Friedrich Gauss.
- Introduces negative numbers (1st),
  Rules for addition/ subtraction of positive and negative numbers.
  Brahmagupta (598-665) came up with the idea of negative numbers in India and Bombelli (1572) in Europe.

Summary of Chinese Mathematics i

- Shang - simple math, Oracle bones
- I Ching influenced Zhou Dynasty - use of hexagrams, binary (Leibniz).
- Decimal system since Shang Dynasty. 1st to use negative numbers.
- Suan shu shu, *A Book on Numbers and Computation*, 202-186 BCE.
  190 Bamboo strips, Found in 1984.
  Roots by False Position and Systems of equations.
- Zhoubi suanjing, *Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*, 100 BCE-100 CE.
  Gougu Thm. [“Around 1100 BC, the Western Zhou period, the ancient Chinese mathematician, Shanggao, first described the Gougu Theorem. ”]
- Jiuzhang Suanshu, *The Nine Chapters on the Mathematical Art.*
- Sun Zi (400-460) *Sunzi suanjing* (Sun Zi’s Mathematical Manual) - Chinese Remainder Thm, Diophatine equations.

Summary of Chinese Mathematics ii

- Zhang Qiujian (430-490) - Manual, Sum arithmetic series, systems of 2 eqns and 3 unknowns.
- Before Han Dynasty - Addition. Subtraction, Multiplication, Division
- After Han Dynasty - Square roots and cube roots.
- Liu Hong (129-210) Calendar, Motion of moon.
- Computing π
    - Liu Xin (d. 23 AD), $\pi \approx 3.1457$.
    - Zhang Heng (78–139), $\pi \approx 3.1724, 3.162$ using $\sqrt{10}$.
    - Liu Hui (3rd century), commented on the Nine Chapters, $\pi = 3.14159$ from 96,192-gon, Exhaustion for circles, Gave method for V_{cyl} (Cavalieri's Principle).
    - Zu Chongzhi (5th century), Mathematical astronomy - Da Ming Li calendar. $\pi = 3.141592$ using 12,288-gon, Remained the most accurate value almost 1000 years. $\pi \approx \frac{355}{113}$ Gave method (Cavalieri's principle) for V_{sphere}.

Summary of Chinese Mathematics iii

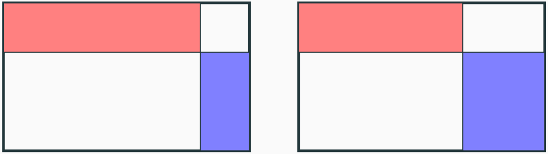
- Liu Zhuo (544-610) - quadratic interpolation.
- Yi Xing (683-727) Tangent table.
- Qin Jiushao (1202–1261) - Treatise of 81 problems
  Up to 10th degree eqns, Chinese Remainder Theorem, Euclidean Algorithm.
- Li Chih (1192-1297) 12 Chapters, 120 problems - Right triangles with circles inscribed/superscribed, geometric problems via algebra.
- Yan Hui (1238–1298), magic squares
- Zhu Shijie (1260 - 1320) - higher degree equations, binomial coefficients, uses zero digit.

One needs to view the methods from their point of view and not ours. In some cases, they provide a simpler "proof by picture."

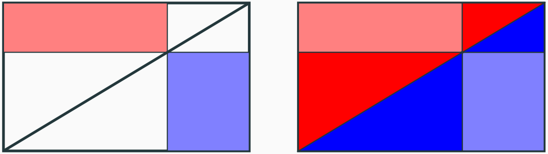
Next: The In-Out Complementary Principle.

In-Out Complementary Method

When do the red and blue rectangles have the same area?



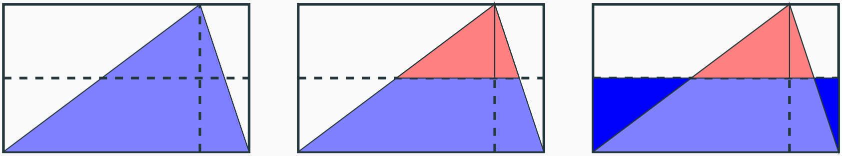
What does this suggest to you?



The case of "Equals subtracted from equals are equal."

Area of a Triangle

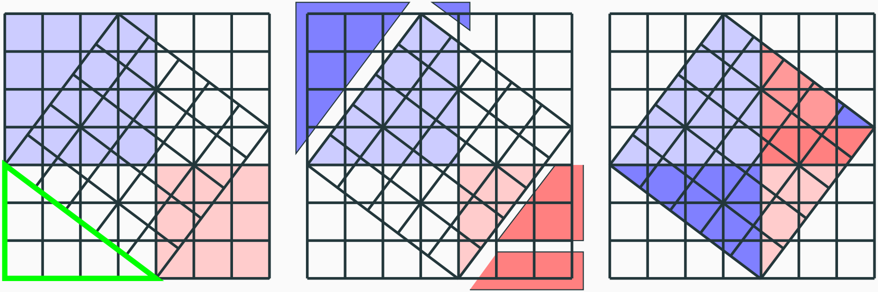
How would you prove that the area of a triangle is $A = \frac{1}{2}bh$?



Move the red "Out" triangles in the middle block to form the blue "In" triangles. The proof should be obvious at this point.

Gōugǔ Theorem

- Gǔ = vertical gauge.
- Gōu = shadow.
- Xián = hypotenuse.



According to J.W. Dauben, *Int. J. of Eng. Sci.* 36 (1998), Liu Hui explains, "The Gou-square is the red square, the Gu -square is the blue square. Putting pieces inside and outside according to their type will complement each other, then the rest (of the pieces) do not move. Composing the Xian-square, taking the square root will be Xian."

Pascal's Triangle

Known by early Chinese mathematicians:

- Systems of linear equations
- Chinese Remainder Theorem
- Square roots
- Pythagorean Theorem
- Euclidean algorithm
- Pascal's Triangle
  - Typical term, $a^{n-k}b^k$, $k = 0, 1, \ldots, n$.
  - What is the coefficient?

$$
\begin{aligned}
(a+b)^0 &= 1\\
(a+b)^1 &= 1a + 1b\\
(a+b)^2 &= a^2 + 2ab + b^2\\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3\\
(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4
\end{aligned}
$$

Figure 3: Binomial Expansion, $(a+b)^n = \sum_{k=0}^{n} C_{n,k} a^{n-k} b^k, \; n = 0, 1, \ldots.$

Yáng Huī, (ca. 1238–1298) presented Jiǎ Xiàn's (ca. 1010–1070) triangle. Used for extracting roots.
Blaise Pascal (1623–1662)

Pascal's Triangle

$$
\begin{aligned}
&1 &&= 1\\
&1+1 &&= 2\\
\text{Sum each row:}\quad &1+2+1 &&= 4\\
\text{Sum} = 2^n.\quad &1+3+3+1 &&= 8\\
&1+4+6+4+1 &&= 16\\
&1+5+10+10+5+1 &&= 32
\end{aligned}
$$

Figure 4: Pascal's Triangle, $C_{n,k} = \binom{n}{k} \equiv \frac{n!}{(n-k)!k!}$

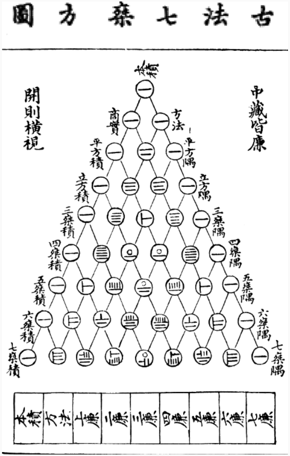


Figure 5: Jia Xian triangle published in 1303 by Zhu Shijie

Euclidean Algorithm Example

Example
One can only go 23 units or 79 units left or right. How many of each would it take to go from 0 to 1?

- $79m + 23n = 1.$
- Use Euclidean Algorithm

$$\begin{aligned} 79 &= 3 \cdot 23 + 10 \\ 23 &= 2 \cdot 10 + 3 \\ 10 &= 3 \cdot 3 + 1 \\ 3 &= 3 \cdot 1 \end{aligned}$$

In Reverse:

$$\begin{aligned} 1 &= 10 - 3 \cdot 3 \\ &= 10 - 3(23 - 2 \cdot 10) \\ &= 7 \cdot 10 - 3 \cdot 23 \\ &= 7 \cdot (79 - 3 \cdot 23) - 3 \cdot 23 \\ &= 7 \cdot 79 - 24 \cdot 23 \end{aligned}$$

Thus, $m = 7, n = -24.$

Chinese Remainder Theorem

The Chinese remainder theorem: If one knows the remainders of the Euclidean division of an integer x by several integers, then one can determine uniquely the remainder of the division of x by the product of these integers, assuming the divisors are pairwise coprime. Earliest - Sun-tzu in *Sunzi Suanjing*.

If $p_1, p_2, \ldots, p_n$ are relatively prime, then

$$\begin{aligned} x &= r_1 \pmod{p_1} \\ x &= r_2 \pmod{p_2} \\ &\vdots \\ x &= r_n \pmod{p_n} \end{aligned}$$

always has a solution.

Chinese Remainder Theorem Example

Example

$$\begin{aligned} x &= 2 \pmod 3 \\ x &= 3 \pmod 5 \\ x &= 2 \pmod 7 \end{aligned}$$

First equation means $x = 3n + 2$. Insert into second:

$$\begin{aligned} 3n + 2 &= 3 \pmod 5 \\ 3n &= 1 \pmod 5 \\ 3n &= 6 \pmod 5 \\ n &= 2 \pmod 5 \end{aligned}$$

So,

$$\begin{aligned} x &= 3n + 2 \\ &= 3(5m + 2) \\ &= 15m + 8. \end{aligned}$$

From third equation

$$\begin{aligned} 15m + 8 &= 2 \pmod 7 \\ 15m + 1 &= 2 \pmod 7 \\ 15m &= 1 \pmod 7 \\ 15m &= 15 \pmod 7 \\ m &= 1 \pmod 7 \end{aligned}$$

Therefore, $m = 7k + 1$ and $x = 105k + 23$.

Indian Mathematics (500-1200)

- Major mathematicians
  - Aryabhata (476-550?)
  - Bhaskara I (600-680)
  - Brahmagupta (598-668)
  - Bhaskara II (1114-1185)
  - Madhava (1350-1425)
- Contributions
  - Algebra
  - Geometry
  - Trigonometry
  - Spherical trigonometry
  - Diophantine Equations
  - Mathematical astronomy
  - Place-value decimal system

Brahmagupta:
$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
$s = \frac{1}{2}(a+b+c+d)$ is semiperimeter

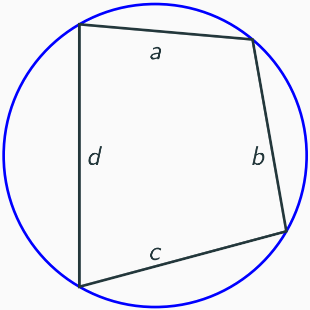


Figure 6: Cyclic Quadrilaterals

Aryabhata (476-550)

- Major work, *Aryabhatiya*, mathematics and astronomy,

  arithmetic, algebra,
  plane trigonometry,
  spherical trigonometry.
  continued fractions,
  quadratic equations,
  sums of power series, and
  table of sines.
- 108 verses, 13 introductory verses
- Relativity of motion
- *Arya-siddhanta*,

  Astronomical computations
  Astronomical instruments

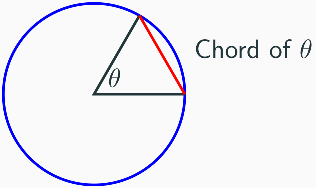
Figure 7: Aryabhata on the grounds of IUCAA, Pune.

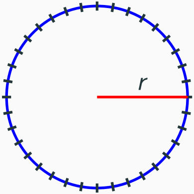
Table of Sines

- Introduction of sine
- Aryabhata's sine table
- Based on half chords
  vs Hipparchus, Menlaus, Ptolemy.
- Also, provided differences
- From Babylonians, base 60
  degrees, minutes, seconds
- Circumference $= 21600'$.
- Aryabhata, $\pi = 3.1416$
- Bhaskara I approximation

$$\sin\theta \approx \frac{4\theta(180-\theta)}{40500-\theta(180-\theta)}.$$

- Mādhava's - more accurate.





$$360^o = 21600'$$

$$r = \frac{21600}{3.1416} \approx 3438$$

Pell's Equation, $x^2 - Ny^2 = 1$, N Nonsquare

Brahmagupta (628): *samasa*, Method of Composition

$$(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2 + Ny_1y_2)^2 - N(x_1y_2 + x_2y_1)^2$$

If $x_1^2 - Ny_1^2 = k_1$ and $x_2^2 - Ny_2^2 = k_2$, then

$$\begin{aligned} x &= x_1x_2 + Ny_1y_2 \\ y &= x_1y_2 + x_2y_1 \end{aligned}$$

solves $x^2 - Ny^2 = k_1k_2$.

This gives a composition of triples, (x_1, y_1, k_1) and (x_2, y_2, k_2) to give (x, y, k_1k_2).

Example (Brahmagupta) $x^2 - 92y^2 = 1$.

Note: $10^2 - 92(1)^2 = 8$. Thus, triple $= (10, 1, 8)$.

Pell's Equation (cont'd)

- $10^2 - 92(1)^2 = 8 \to (10, 1, 8)$.
- Compose $(10, 1, 8)$ with itself. $(10 \cdot 10 + 92 \cdot 1 \cdot 1, 10 \cdot 1 + 1 \cdot 10, 8 \cdot 8) = (192, 20, 64)$
- or, $192^2 - 92(20)^2 = 64$
  $24^2 - 92\left(\frac{5}{2}\right) = 1$
- Compose $(24, \frac{5}{2}, 1)$ with itself: $(1151, 120, 1)$.
- Bhaskara II (1150) - cyclic process always works - *chakravala*.
- Proved by Lagrange (1768)
  $gcd(a, b) = 1,\ a^2 - Nb^2 = k$.
- Compose (a, b, k) with $(m, 1, m^2 - N)$ gives $(am + Nb, a + bm, k(m^2 - N))$.
- Rescale

$$\left(\frac{am + Nb}{k}, \frac{a + bm}{k}, m^2 - N\right)$$

- Fermat (1657), $x^2 - 61y^2 = 1$,
  $x = 1766319049$,
  $y = 226153980$.

Japanese Mathematics (Wasan)

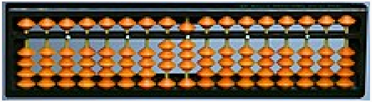
- Developed in Edo Period (1603-1867) rule of the Tokugawa shogunate.
- Economic growth, strict social order, isolationist foreign policies, a stable population, perpetual peace, and popular enjoyment of arts and culture.
- Foreign trade restrictions
- Mathematics for taxes
- Used Chinese counting rods.
- Imported Chinese texts.
- Sangi rods, adopted from Chinese rods, called saunzi.

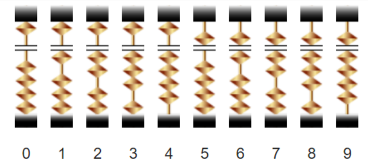
Second period (552-1600) - Influx of Chinese learning - first through Korea.

Figure 8: Seki Takakazu.

The Soroban

- Abacus developed in Japan.
- Derived from ancient Chinese **suanpan**,
- Imported to Japan, 14th century.
- go-dama = 5, ichi-dama = 1.
- Addition, substraction, multiplication, division.





Yoshida Mitsuyoshi (1598–1672), *Jinkōki,* (1627) oldest existing Japanese math text, subject of **soroban arithmetic**, incl. square and cube roots.

Student of Kambei Mori, **first Japanese mathematician** with students Imamura Chishō, and Takahara Kisshu ("Three Arithimeticians").

Seki Takakazu (1642–1708), infinitesimal calculus and Diophantine equations, **"Japanese Newton."**

The Rise of Islam

- Mohammad was born in Mecca (570)
- Began preaching in Mecca, Escaped to Medina (622), later returned (629).
- Under Umar, Son of Al-Khattab, the empire spread ruled the Sasanian Empire and more than two-thirds of the Byzantine Empire
- Eventually from India to Spain. Main centers at Baghdad and Cordova.
- Caliph Al-Ma'mun created Bait Al-hikma (House of Wisdom), Around 800 CE.
- Arabic became the common language.
- Translation of Greek and Hindu to Arabic (Euclid et al.) began with Al-Mansor, founder of Baghdad, grandfather of Al-Ma'mun.

Islamic Mathematics

- Caliph Al-Ma'mun appointed Al-Khwarizmi (780-850) court astronomer.
- His book, *Hisab al-jabr w'al-muqabala*, Solve linear or quadratic, 6 forms. Terms: Algebra and Algorithm.
- Al-Kindi (801-873) Arithmetic, 11 texts.
- Al-Bhattini (850-929)
  - Trigonometry, 1st Cotangent table.
- Thabit ibn Qurra (836-901) Theory of Numbers.
- Al-Kuhi (c940-1000) Archimedean and Apollonian math. Equations of degree > 2.
- Arabic numerals 1st in a book 874 and zero 2 yrs before Hindus.

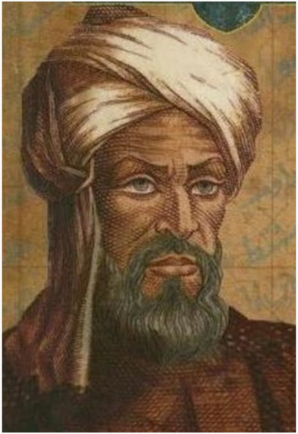


Figure 9: Al-Khwarizmi

International Year of Light 2015

Ibn al-Haytham's scientific method

Hasan Ibn al-Haytham (Latinized Alhazen)

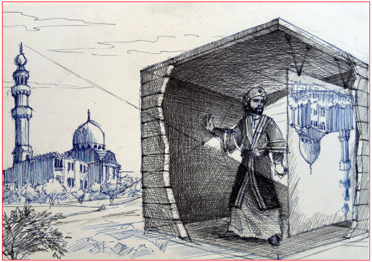
During the International Year of Light 2015, Ibn al-Haytham was celebrated at UNESCO as a pioneer of modern optics. He was a forerunner to Galileo as a physicist, almost five centuries earlier, according to Prof. S.M. Razaullah Ansari (India). Also known as Alhazen, this brilliant Arab scholar from the 10th – 11th century, made significant contributions to the principles of optics, astronomy and mathematics, and developed his own methodology: experimentation as another mode of proving the basic hypothesis or premise.

by Shaikh Mohammad Razaullah Ansari

Abū Ali al-Ḥasan Ibn al-Haytham al-Baṣrī (965-1040), known in European Middle Ages by the name of Alhazen, was called among Arab scholars as 'Second Ptolemy' (Baṭlamyūs Thānī). He was actually a scholar of many disciplines: Mathematics, physics, mechanics, astronomy, philosophy and medicine. He was one of the senior most member of the Muslim scholars' trio during 10th -11th centuries, the other two were al-Bīrūnī (973-1048) and Ibn Sīnā (980–1037).

From Basra, Ibn al- Haytham shifted to Cairo, where the Fatimid Caliph al-Ḥākim had invited him. The Caliph was a great patron of scientist-scholars, he got built an observatory for the astronomer Ibn Yūnus (d.1009) and he founded a library Dār al–'Ilm, whose fame almost equaled that of its precursor at Baghdad, Bayt al- Ḥikma(the House of Wisdom), established by the Abbasid Caliph al-Mā'mūn (reigned 813 – 833).

Ibn al-Haytham was a prolific writer. According to his own testimony, he wrote 25 works on mathematical sciences, 44 works on (Aristotelian) physics and metaphysics, also on meteorology and psychology. Moreover, his autobiographical sketch indicates clearly that he studied very thoroughly Aristotle's (natural) philosophy, logic and metaphysics of which he gave a concise account.



Ibn al-Haytham's pinhole camera and Al-Farisi's Rainbow

Later Islamic Mathematicians

- Ibn Al-Haytham (965-1039) Studied optics and visual perception.
  Used Euclid and Apollonius for Reflection, refraction, spherical mirrors, rainbows, eclipses, shadows.
  Optics works in Europe 12-13th century.
  Also, tackled 5th postulate.
- Golden Age: 9-10th centuries.
- Al-Karkhi (d. 1019-1029) Arithmetic, $\sum_n n^k$, $k = 1, 2, 3$.
- Omar Khayyam (1048-1131) studied cubic equations, and the intersection of conics, Khayyam's triangle, parallel postulate.
- Al-Kashi (1380-1429), fractions, π to 16 places, Law of Cosines.

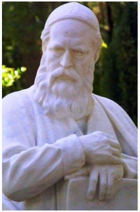
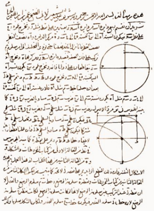
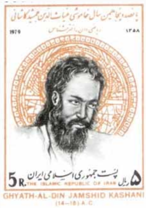




Figure 10: Omar Khayyam and Jamshīd al-Kashi stamp.

Rise of European Mathematics

- Fall of the Roman Empire
- Middle Ages, Medieval Period, 5th to the 15th century.
- Byzantine Empire (330-1453) - Church split,
- Preservation of Greek works.
- Al'Khwarizmi's work and Euclid translated into Latin.
- Crusades (1095-1291), The Plague (1347 to 1351).
- Mongols destroyed Islamic empire 1258.
- Johannes Gutenberg' printing press, 1440.
- Renaissance (1400-1600) and the Age of Discovery.
- Questioning of Aristotle
- Church of England (1534), Protestant vs Roman Catholic

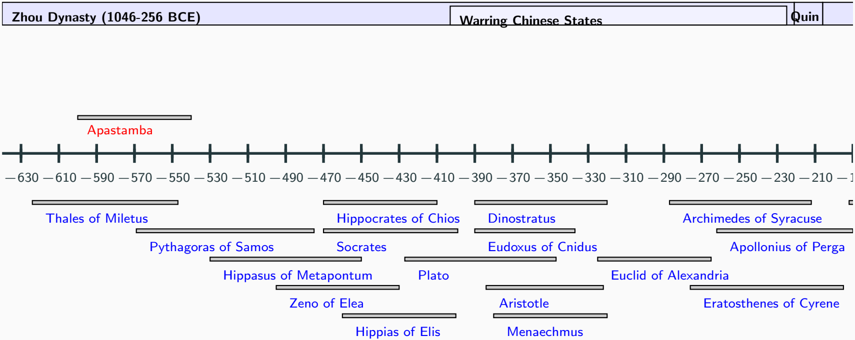
Medieval Mathematicians

- Leonardo of Pisa (Fibonacci) (1200).
- Nicole Oresme (1323-1382), coordinate geometry, fractional exponents, infinite series.
- Johann Müller Regiomontatus (1436-1476), separated trigonometry from astronomy.
- And others:

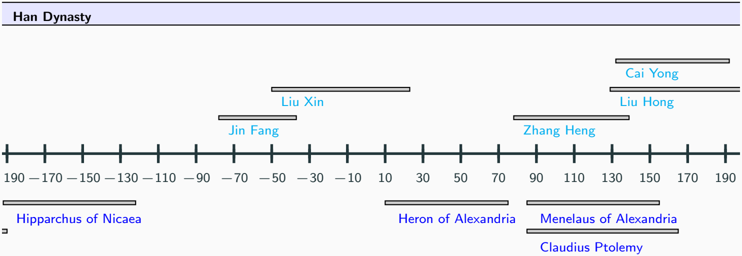
Roger Bacon (1214-1292)	William of Ockham (1288-1348)
Filippo Brunelleschi (1377-1446)	Leone Battista Alberti (1404-1472)
Nicholas of Cusa (1401-1464)	Piero della Francesca (1420 - 1492)
Leonardo da Vinci (1452-1519)	Luca Pacioli (1445-1517)
Nicolaus Copernicus (1473-1543)	Scipione del Ferro (1465-1526)

- Rise of European Mathematics .. beginning in Italy.

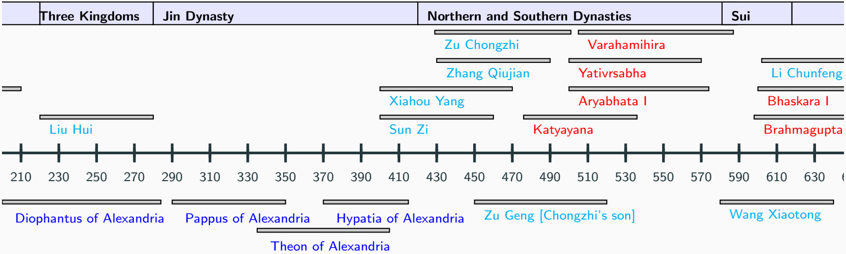
Timeline of Ancient Mathematicians i



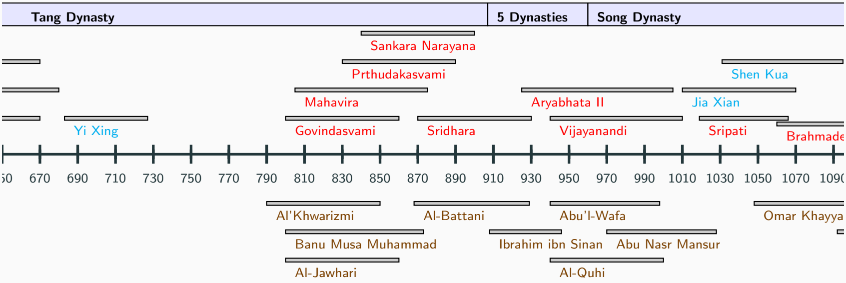
Timeline of Ancient Mathematicians ii



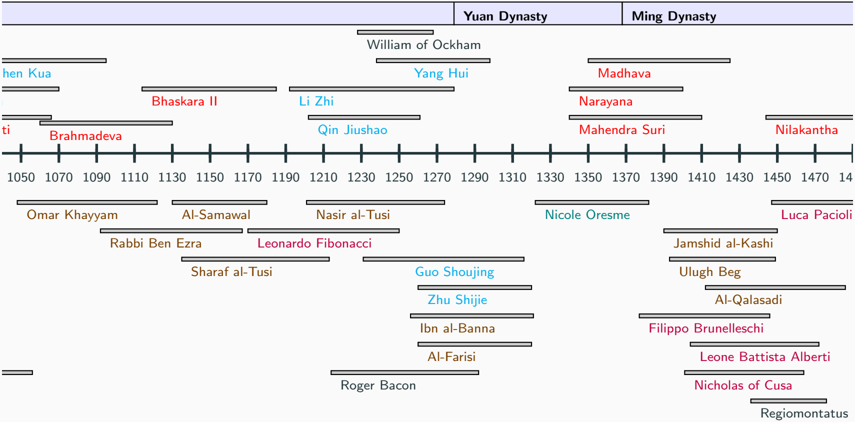
Timeline of Ancient Mathematicians iii



Timeline of Ancient Mathematicians iv



Timeline of Ancient Mathematicians v



Chinese Mathematical Classics

The Suàn shù shū (A Book on Numbers and Computations) is written on nearly 200 bamboo slips. The earliest known mathematical work in Chin, recently discovered in a tomb, dated to the early second century BCE.

The Jiu zhang suan shu (Nine Chapters on the Art of Mathematics) known from an imperfect copy printed in the Southern Song dynasty (1213 CE)

The Shi bu suan jing (Ten Books of Mathematical Classics)

- Zhoubi suanjing (Zhou Shadow Gauge Manual)
- Jiuzhang suanshu (Nine Chapters on the Mathematical Art)
- Haidao suanjing (Sea Island Mathematical Manual)
- Sunzi suanjing (Sun Zi's Mathematical Manual)
- Wucao suanjing (Mathematical Manual of the Five Administrative Departments)
- Xiahou Yang suanjing (Xiahou Yang's Mathematical Manual)
- Zhang Qiujian suanjing (Zhang Qiujian's Mathematical Manual)
- Wujing suanshu (Arithmetic methods in the Five Classics)
- Jigu suanjing (Continuation of Ancient Mathematics)
- Shushu jiyi (Notes on Traditions of Arithmetic Methods)
- Zhui shu (Method of Interpolation)
- Sandeng shu (Art of the Three Degrees; Notation of Large Numbers)