Introduction to the History of Mathematics

Fall 2023 - R. L. Herman

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Early Civilizations - Babylonian, Egyptian, Chinese, Indian, Islamic

Renaissance Mathematics - 15th-16th Centuries

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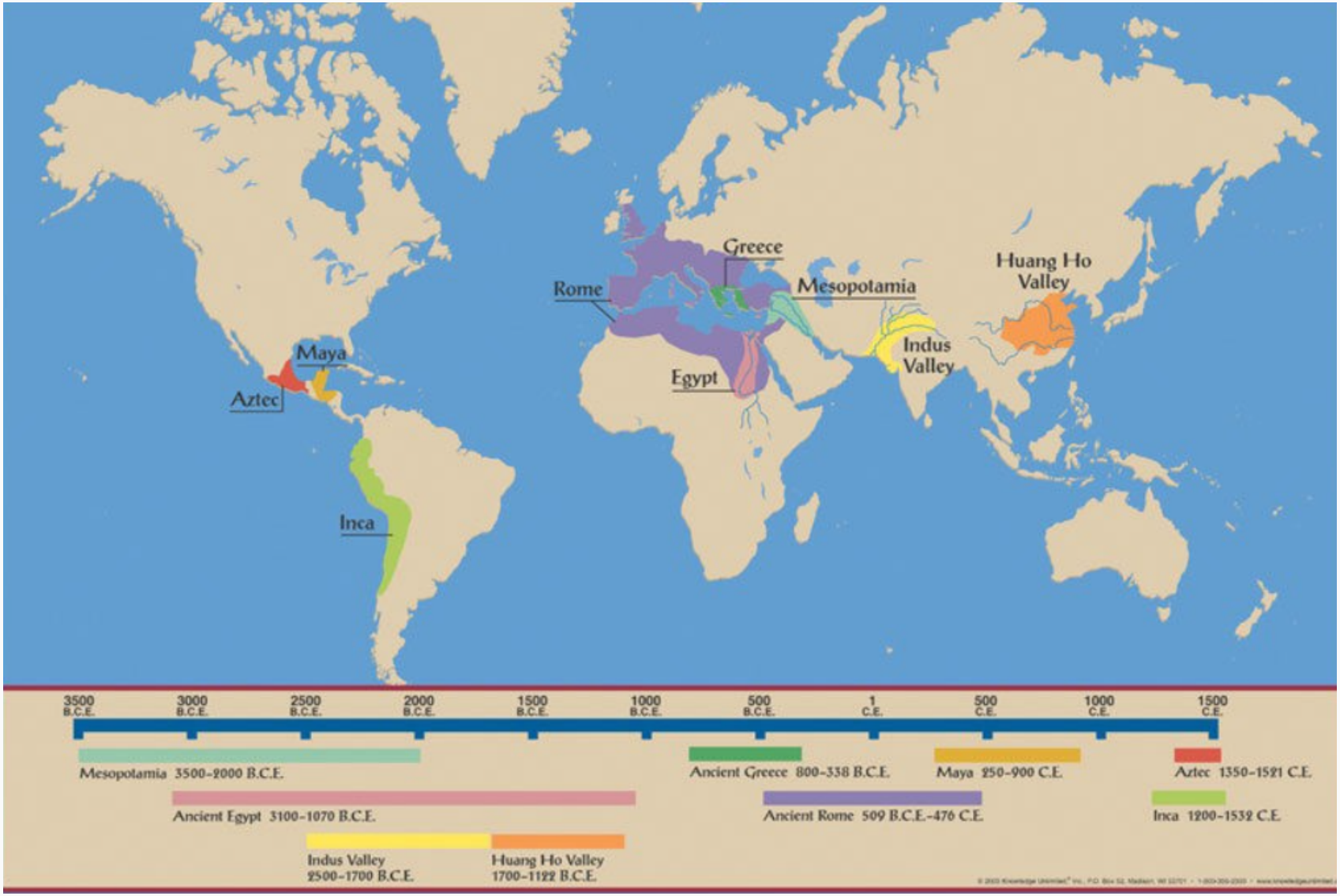
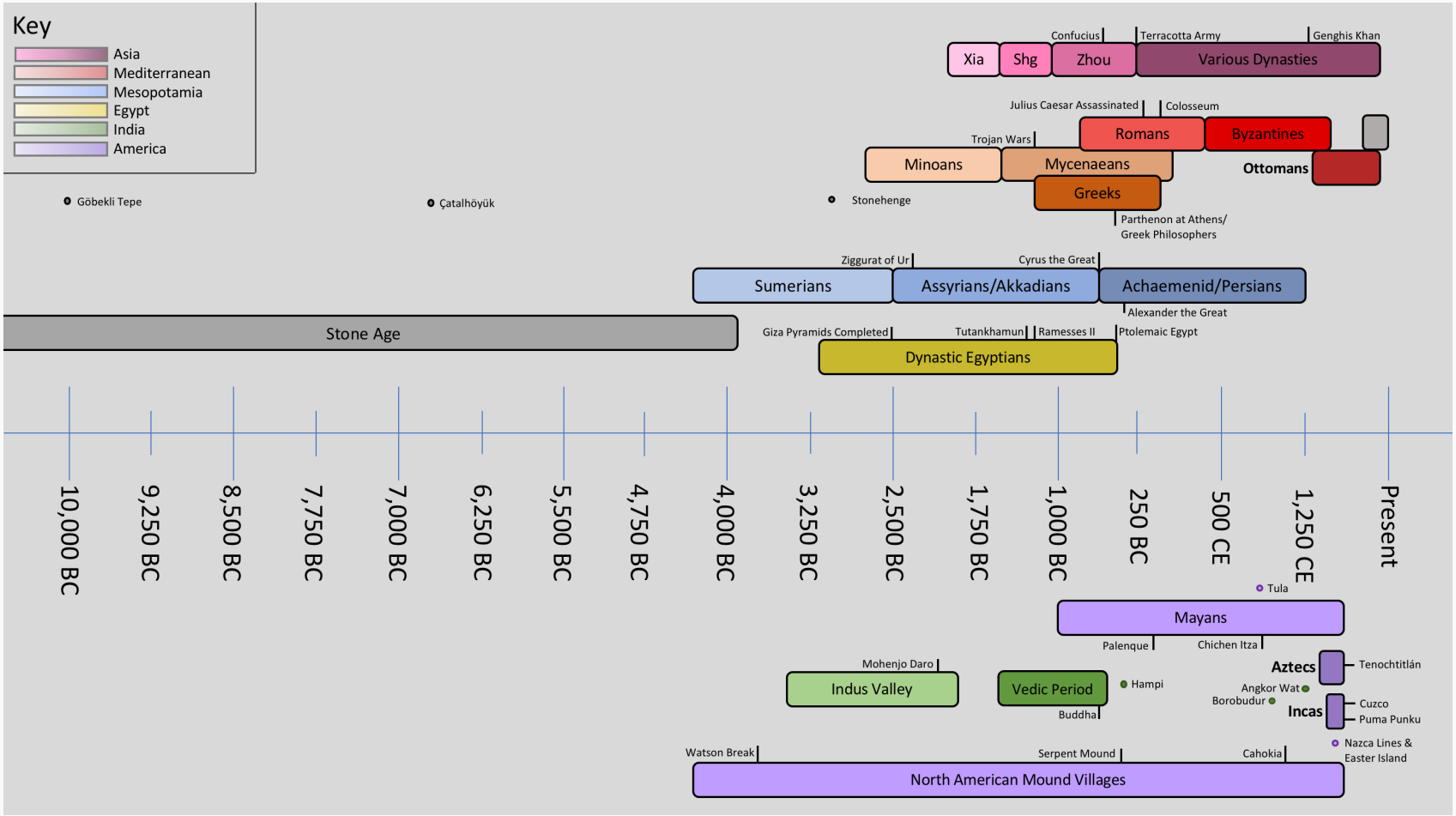
Civilizations - How Did Mathematics Develop?



A Civilization Timeline



Some Early Civilizations

- Egypt (3150-30 BCE)
- Mesopotamia (3100-539 BCE)
- Chinese (1766 BCE-220 CE)
- Indian Mathematics (500-1200)
- Mayan Mathematics (250-900)
- Aztecs and Incans (1345-1560)

Arithmetic, Geometry, No proofs.
Problems were practical or recreational.

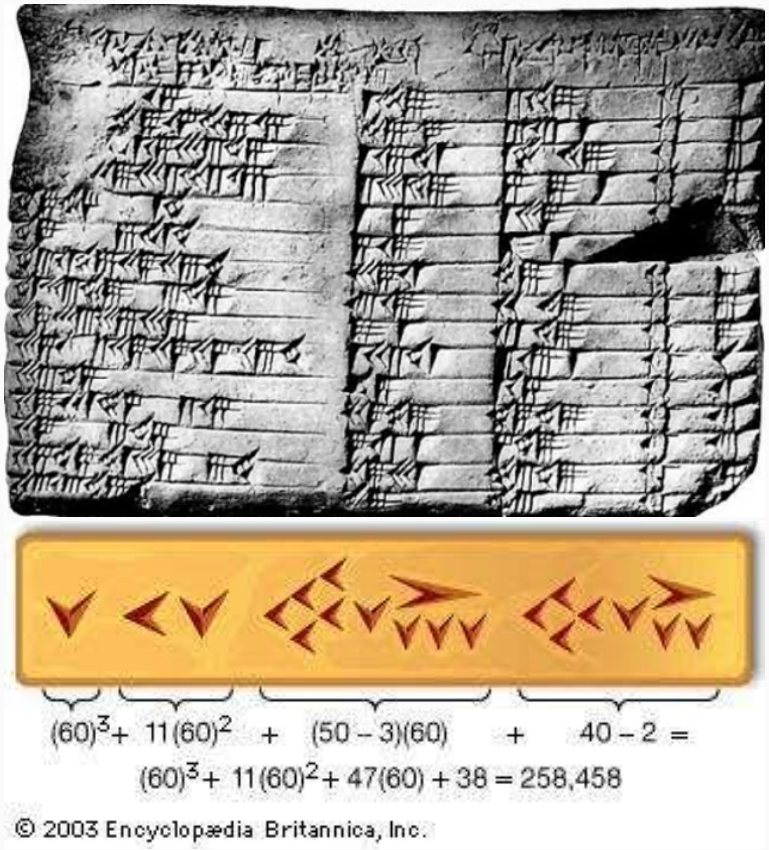


Figure 1: Babylonian Math - Base 60

Greek Civilization

- Deductive Reasoning
  - Definitions, Axioms
  - Propositions via logic
- Geometry, Trigonometry, Astronomy, Numbers, Conics
- Thales (624-546 BCE)
- Pythagoras (6th Century BCE)
- Euclid (4th Century BCE)
  Elements - geometry, numbers
- Archimedes (3rd Century BCE)
- Apollonius (2nd Century BCE)
- Heron (10-70), Diophantus (200-284), Pappas (290-350), Hypatia (370-415)

Figure 2: Euclid

Chinese and Indian

- Chinese Mathematics
  1300 BCE - 1800 CE
  - Pythagorean Thm
  - π estimates
  - Volumes, Applications
  - Pascal's Triangle
  - Chinese Remainder Thm
- Indian Mathematics 1200 BCE,
  mostly 500-1200 CE
  - Geometry
  - Trigonometry
  - Power series
  - Astronomy
  - π estimates
  - Number system, 0
  - Pell's Equation

Figure 3: Liu-Hong

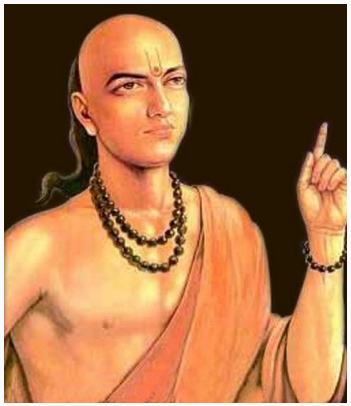


Figure 4: Aryabhatta

Middle Eastern Mathematics - 700-1200 CE

European Dark Ages - 400-1200 CE

- Founding of Islam - 7th Century
- Islamic mathematicians preserve/translate Greek/Asian mathematics into Arabic
- Arabic Numerals by 1000 CE
- Persian mathematicians
  - al-Khwarizmi (780-850)
    - Algebra (al-Jabr)
  - Omar Khayyam (1048-1131)
    - geometric solution of cubic

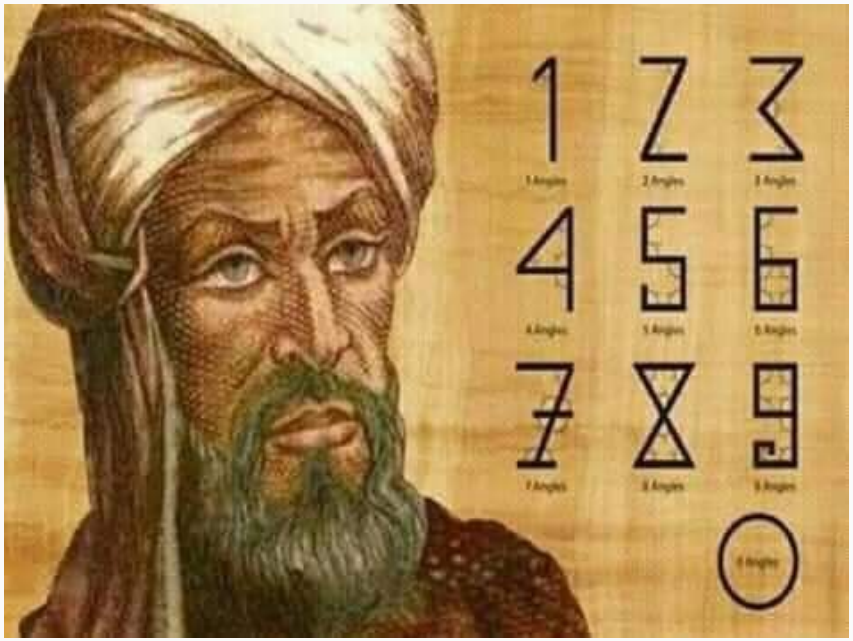
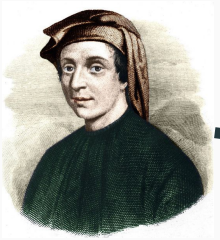


Figure 5: al-Khwarizmi

Around 10th Century - Middle Eastern Mathematics brought to Spain.

It takes 300 years to accept Hindu-Arabic numerals. - Fibonacci - 1202

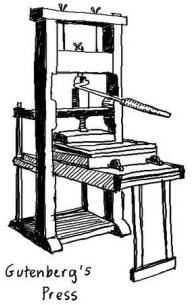
The Renaissance



Leonardo of Pisa (1175-1250)

Black Death (1347-1351)

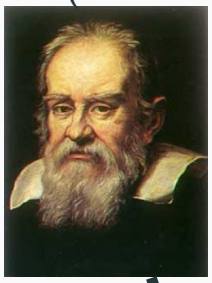
The Renaissance 1400-1600



Printing Press (1440)

Luca Pacioli (1445-1517)

Galileo (1564-1642)



Giovanni di Medici
Banking (1397)

Perspective in Art (1377-1492)

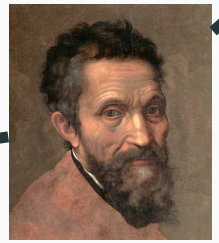
Bruneschelli, Alberti, Piero della Francesca

Leonardo da Vinci (1452-1519)

Reformation (1517)

Cardano (1501-1576)

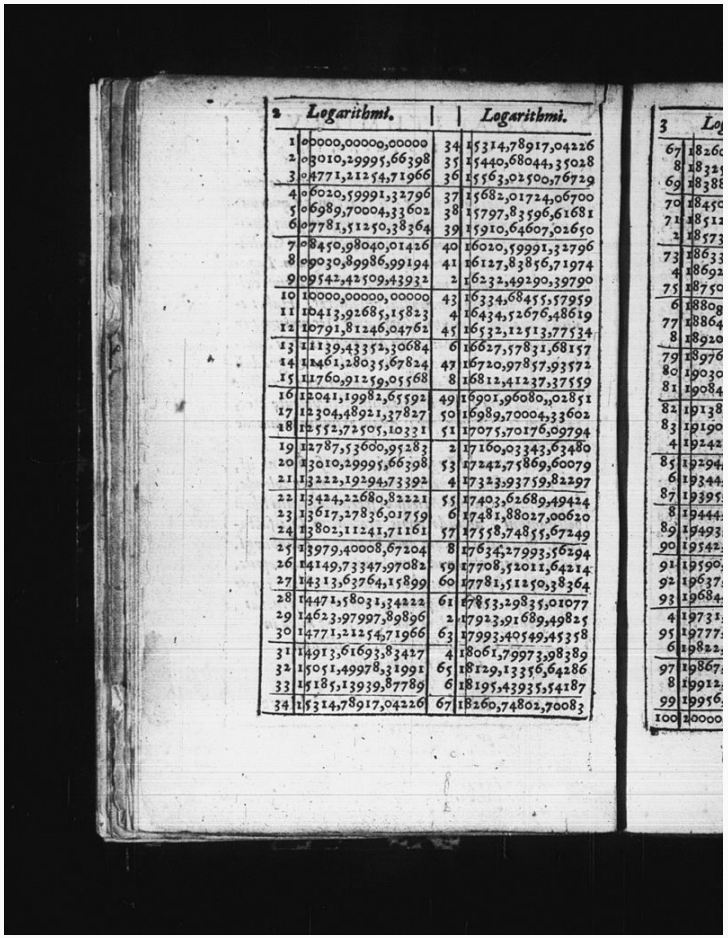
Tartaglia (1500-1557)



Michelangelo (1475-1564)

Beyond Numerals

- Fractions 4000 years ago
- Sexagesimal (base 60) into 17th century
- Decimal (base 10)
  - al-Uqlidisi - (920-980)
  - al-Kashi (1380-1429)
  - Simon Stevin (1548-1620)
- Logarithms
  - John Napier (1550-1617) - used a stange base
  - Henry Briggs (1561-1630)
    - Base 10 Tables
    - 54 square roots of 10 (30 decimal places)
    - Tables - 14 decimal places



2	Logarithmi.		Logarithmi.
1	00000,00000,00000	34	15314,78917,04226
2	03010,29995,66398	35	15440,68044,35028
3	04771,21254,71966	36	15563,02500,76729
4	06020,59991,32796	37	15682,01724,06700
5	06989,70004,33602	38	15797,83596,61681
6	07781,51250,38364	39	15910,64607,02650
7	08450,98040,01426	40	16020,59991,32796
8	09030,89986,99194	41	16127,83856,71974
9	09542,42509,43932	2	16232,49290,39790
10	10000,00000,00000	43	16334,68455,57959
11	10413,92685,15823	4	16434,52676,48619
12	10791,81246,04762	45	16532,12513,77534
13	11139,43352,30684	6	16627,57831,68157
14	11461,28035,67824	47	16720,97857,93572
15	11760,91259,05568	8	16812,41237,37559
16	12041,19982,65592	49	16901,96080,02851
17	12304,48921,37827	50	16989,70004,33602
18	12552,72505,10321	51	17075,70176,09794
19	12787,53600,95283	2	17160,03343,63480
20	13010,29995,66398	53	17242,75869,60079
21	13222,19294,73392	4	17323,93759,82297
22	13424,22680,82221	55	17403,62689,49424
23	13617,27836,01759	6	17481,88027,00620
24	13802,11241,71161	57	17558,74855,67249
25	13979,40008,67204	8	17634,27993,56294
26	14149,73347,97082	59	17708,52011,64214
27	14313,63764,15899	60	17781,51250,38364
28	14471,58031,34222	61	17853,29835,01077
29	14623,97997,89896	2	17923,91689,49825
30	14771,21254,71966	63	17993,40549,45358
31	14913,61693,83427	4	18061,79973,98389
32	15051,49978,31991	65	18129,13356,64286
33	15185,13939,87789	6	18195,43935,54187
34	15314,78917,04226	67	18260,74802,70083

Figure 6: Brigg's Tables

Italian Mathematics - Polynomial Equations, Complex Numbers

- Fibonacci (Leonardo of Pisa) (1170-1250) *Liber Abaci*
- Equation Solving contests
- Solutions of cubic and quartic
  - Depressed cubic
    del Ferro (1465-1526)
  - Cubic and quartic equations
    Tartaglia (1500-1557)
    Cardano (1501-1576)

    Ars Magna

    Ferrari (1522-1565)
- Bombelli (1526-1572)
  - Complex numbers
- Viète (1540-1603)
  Adriaan van Roomen Problem

Figure 7: Cardano and Tartaglia
Fight of the Century!

Unification of Geometry and Algebra

- Symbolic Algebra
  - Rhetorical until 15th century
  - Syncopated/abbrev. - 1500
  - Symbolic algebra developed 16-17th century
- Unification
  - Oresme (1320-1382) - Velocity-time graphs, $\sum \frac{1}{n}$
  - Descartes (1596-1650)
    - Rep. curves by equations
    - Coordinate systems - published *The Method*
    - Made use of variables which can vary continuously - lines.
  - Fermat (1607-1665)
    - Rep. equations by curves

Figure 8: Fermat and Descartes

The Rise of Calculus

- Archimedes - 3rd century BCE
- Kepler (1571-1630)
- Cavalieri (1598-1647)
- Fermat (1607-1665)
- Wallis (1616-1673)
- Pascal (1623-1662)
- Barrow (1630-1677)
- Wren (1632-1723)
- Gregory (1638-1675)
- Newton (1642-1726)
  - *Principia* 1687
- Leibniz (1646-1716)
  - *Notation* $\frac{d}{dx}, \int$

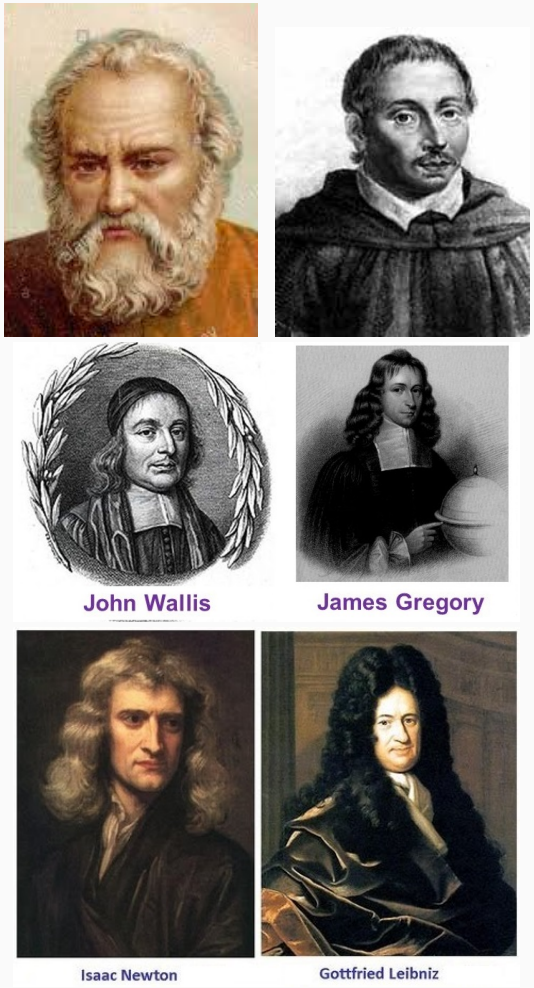
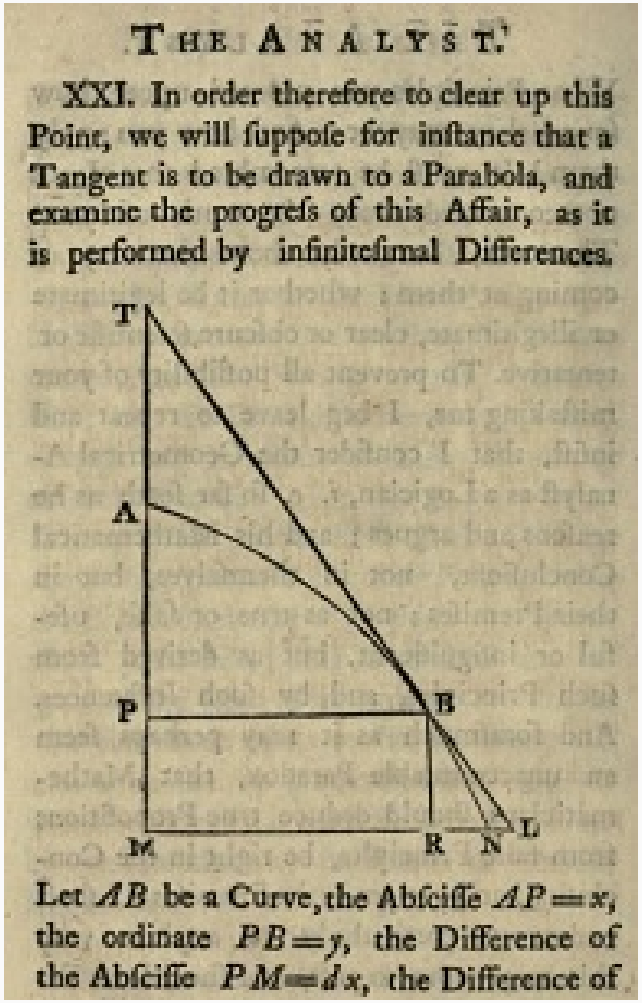


Figure 9: Archimedes, Cavalieri, Wallis, Gregory, Newton, and Leibniz

The Infinitesimal

- Hippasus 500 BCE
  - Pythagorean, $\sqrt{2}$ irrational
- Introduction of Infinitesimals
  - Cavalieri and Torricelli
  - Stevin, Wallis, Harriot
- Religious Critics
  - Era of Copernicus, Galileo
  - Jesuits in Italy, Church bans
  - George Berkeley (1685-1753) *The Analyst*, - *A Discourse Addressed to an Infidel Mathematician*, 1734
  - "Infinitesimals undermine mathematics and rationality"
- Augustin-Louis Cauchy - 1821


THE ANALYST.

XXI. In order therefore to clear up this Point, we will ſuppoſe for inſtance that a Tangent is to be drawn to a Parabola, and examine the progreſs of this Affair, as it is performed by infiniteſimal Differences.

Let AB be a Curve, the Abſciſſe $AP = x$, the ordinate $PB = y$, the Difference of the Abſciſſe $PM = dx$, the Difference of

Figure 10: Berkeley's *The Analyst*

Exploiting Calculus

- Bernoulli Family
- Leonhard Euler (1707-1783)
- Joseph-Louis Lagrange (1736-1813)
- Pierre-Simon Laplace (1749-1827)
- Neptune discovered using math - 1846, Le Verrier (1811-1877)

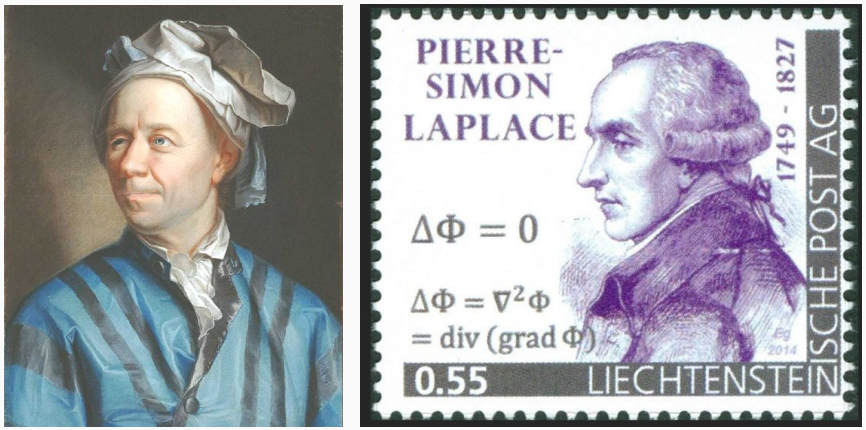


Figure 11: Euler and Laplace

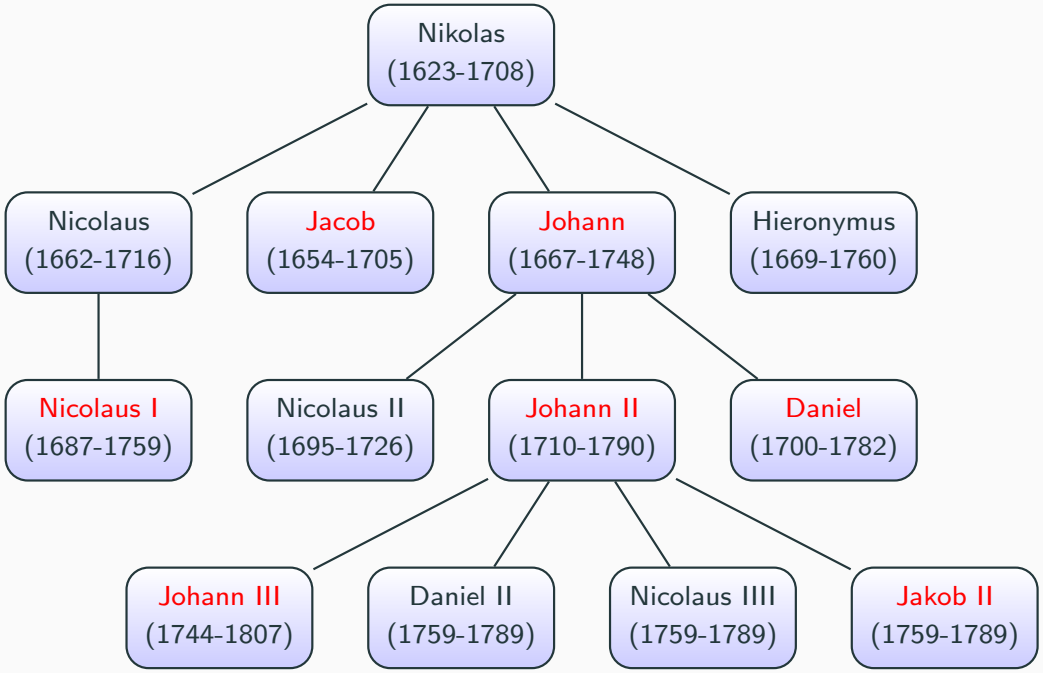
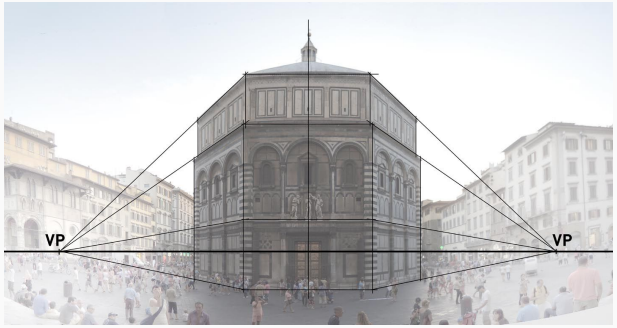


Figure 12: The Bernoulli Family

Evolution of Geometry: Projective Geometry and Topology

- Perspective in Art
  - Bruneschelli (1377-1446)
  - Leon Alberti (1404-1472)
  - Girard Desarges (1591-1661)
- Birth of Topology
  - Euler - Königsberg bridge Geometry without distance
  - Euler Characteristic $\chi = V - E + F$, and classification of surfaces $\chi = 2 - 2g$, genus
- Birth of Knot Theory
  - Gauss - Intertwining curves
  - Scottish physics and Knots - Taits smoke rings.



The Birth of Rigor - 19th Century

Non-Euclidean Geometry

- Parallel Postulate
- Hyperbolic Geometry
  - Nikolai Lobachevsky (1792-1856)
  - Johann Bolyai (1802-1860)
  - Johann Carl Friedrich Gauss (1777-1855)
- Elliptic Geometry
  - Georg Friedrich Bernhard Riemann (1826-1866)
  - Prince of Mathematicians
- By 1870's Euclid in doubt!

Figure 13: Gauss, Lobachevsky, Bolyai

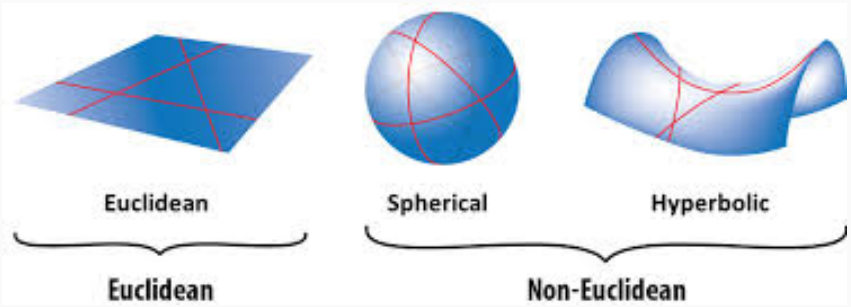


Figure 14: Different Geometries

19th Century Group Theory

The search for the general solution of the quintic.

- Joseph-Louis Lagrange (1736-1813)
- Johann Carl Friedrich Gauss (1777-1855)
- Paola Ruffini (1765-1822) - proof of unsolvability
- Augustin Cauchy (1789-1857)
- Niels Henrik Abel (1802-1829)
- Évariste Galois (1811-1832)
- Arthur Cayley (1821-1895)
- Camille Jordan (1838-1922)

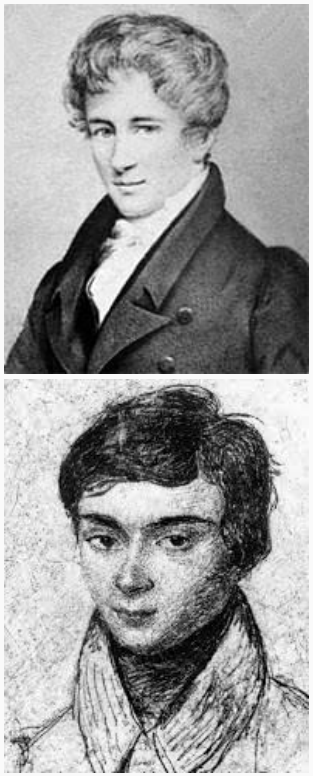


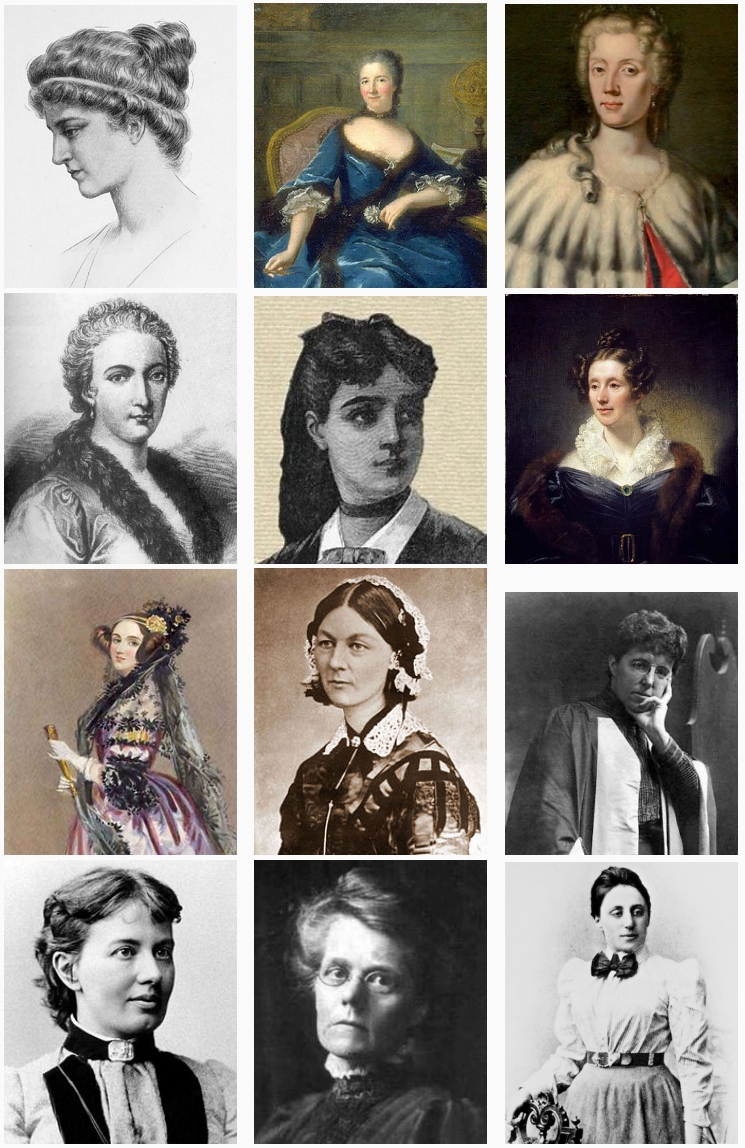
Figure 15: Abel and Galois

Mathematical Physics - Mathematical Physicists

- Joseph-Louis Lagrange (1736-1813)
- Pierre-Simon Laplace (1749-1827)
- Joseph Fourier (1768–1830)
- Siméon Denis Poisson (1781–1840)
- George Green (1793–1841)
- William Hamilton (1805–1865)
- William Thomson (1824–1907), 1st Baron Kelvin
- James Clerk Maxwell (1831–1879)
- J. Willard Gibbs (1839–1903)
- John William Strutt (1842–1919), 3rd Baron Rayleigh
- Oliver Heaviside (1850–1925)

Famous Women Mathematicians Before 1900

- Hypatia of Alexandria (c. 350-415)
- Émilie du Châtelet (1706-1749)
- Laura Bassi (1711-1788)
- Maria Agnesi (1718-1799)
- Sophie Germain (1776-1831)
- Mary Fairfax Somerville (1780-1872)
- Ada Lovelace (1815-1852)
- Florence Nightingale (1820-1910)
- Charlotte Angas Scott (1848-1931)
- Sofia Kovalevskaya (1850-1891)
- Alicia Boole Stott (1860-1940)
- Amalie 'Emmy' Noether (1882-1935)



19th Century Analysis and Set Theory

- Jean-Baptiste Joseph Fourier (1768-1830)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Karl Weierstrass (1815-1897)
- George Boole (1815-1864)
- Georg Friedrich Bernhard Riemann (1826-1866)
- Richard Dedekind (1831-1916)
- Georg Ferdinand Ludwig Philipp Cantor (1845-1918)
  - Founder of set theory
  - Defined infinite sets

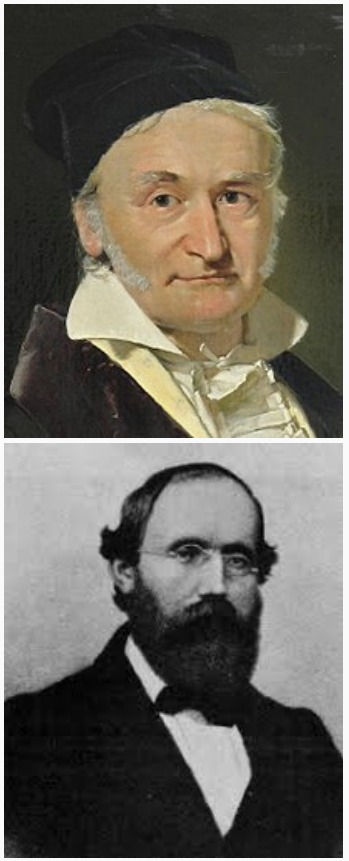


Figure 16: Gauss and Riemann

19th Century Number Theory

- Marie-Sophie Germain (1776-1831)
- Johann Carl Friedrich Gauss (1777-1855)
  Disquisitiones Arithmeticae - 1801
- Adrien-Marie Legendre (1752-1833) and
  Peter Gustav Lejeune Dirichlet (1805-1859) prove
  Fermat's Last Theorem for $n = 5$ in 1825
  - Dirichlet, $n = 14$ in 1832.
- Riemann Hypothesis, distribution of primes - 1832.
- Charles Jean de la Vallée-Poussin and Jacques Hadamard - Prime Number Theorem. 1896
- H. Minkowski: Geometry of Numbers, 1896.

Figure 17: Sophie Germain, Adrien-Marie Legendre

The Modern Era

We stop at the turn of the 20th Century: the evolution of mathematics, set theory, physics revolutions, Bourbaki, Hilbert's 23 Problems.

Explore mathematics prizes: Fields Medal, Abel Prize, Wolf Prize, Millenium Prize.

Other Sites

- Chronology of 20th Century Mathematicians
- Greatest Mathematicians born between 1860 and 1975
- Pictures of Famous 20th Century Mathematicians
- The Story of Math Website

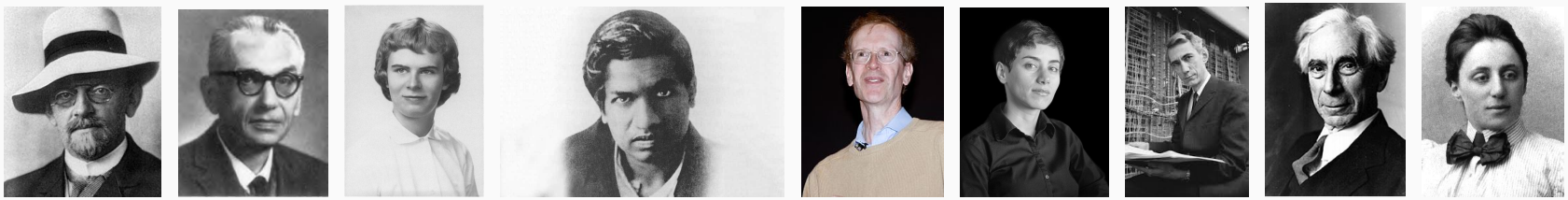


Figure 18: Hilbert, Gödel, Uhlenbeck, Ramanujan, Wiles, Mirzakhani, Shannon, Russell, Noether