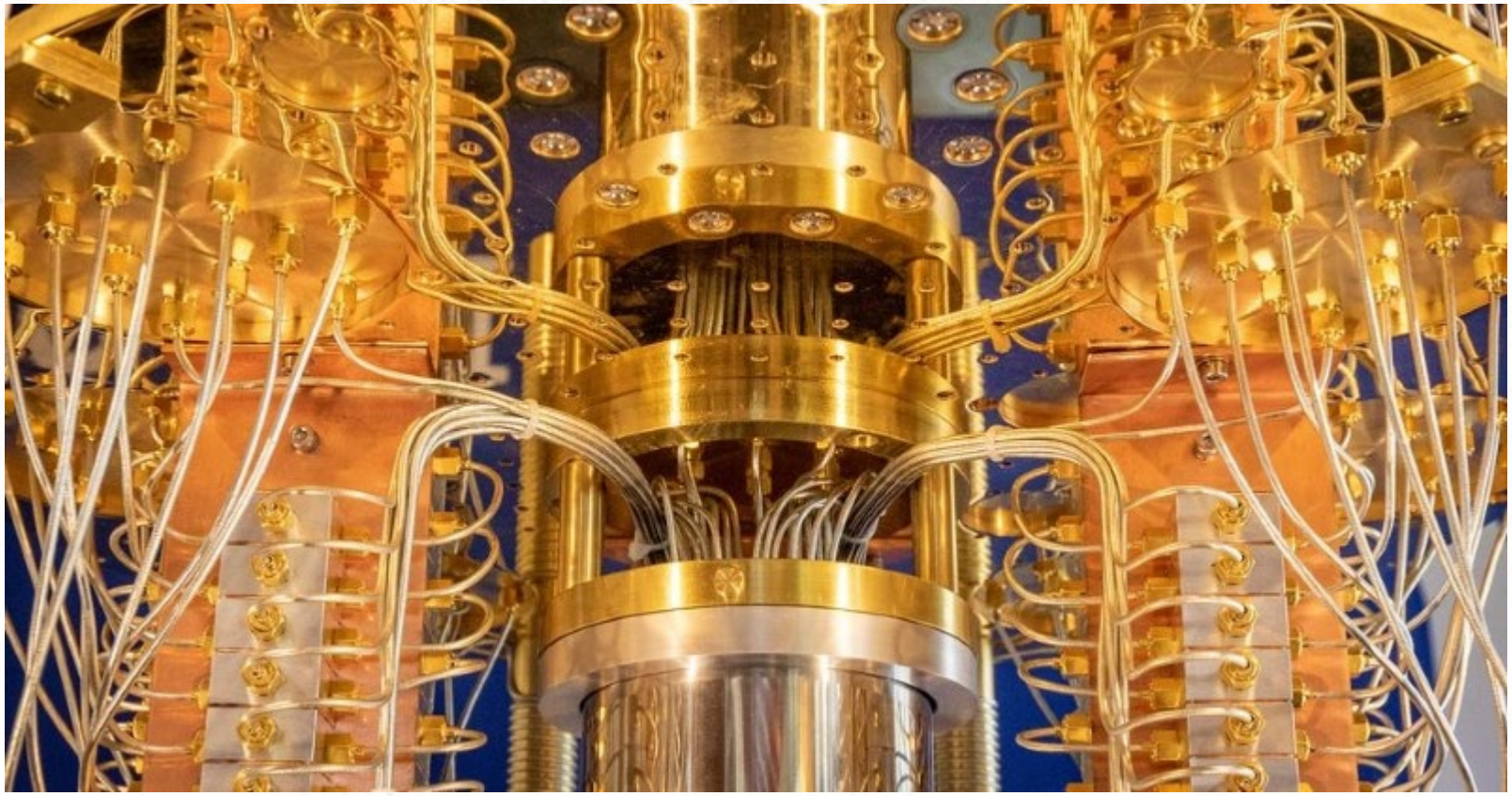
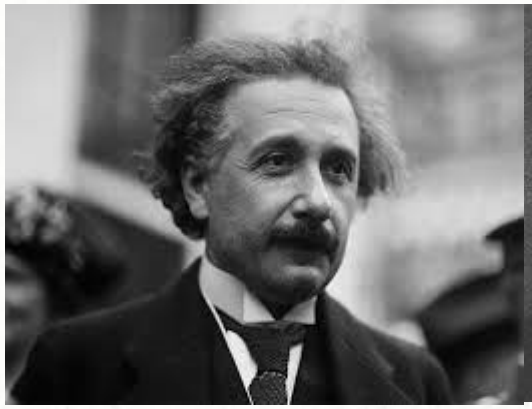
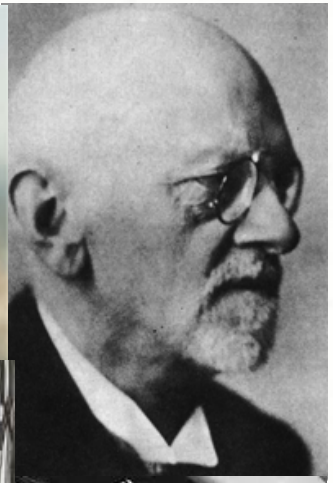
History of Mathematics - 1900s

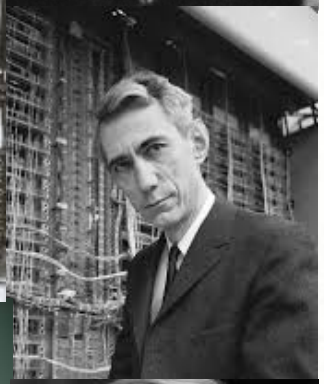
Dr. R. L. Herman, UNCW

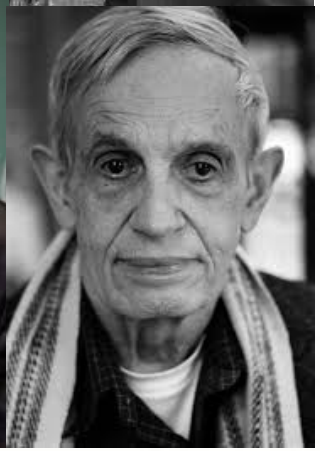
MAT 346, Fall 2023







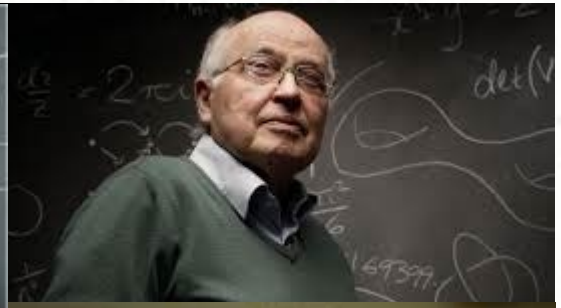



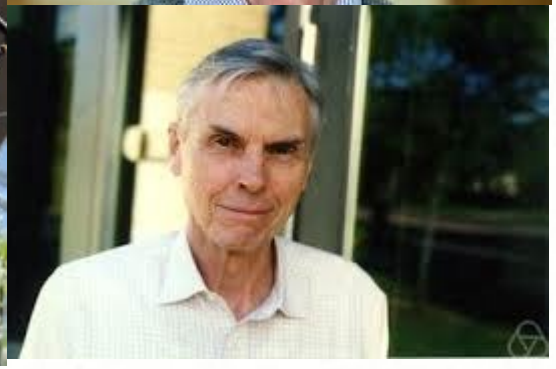
Outline

- The Rise of Rigor
- Set Theory
- N. Bourbaki
- Physics Revolutions
- Hilbert's Problems
- Mathematics Prizes

Mathematics Prizes

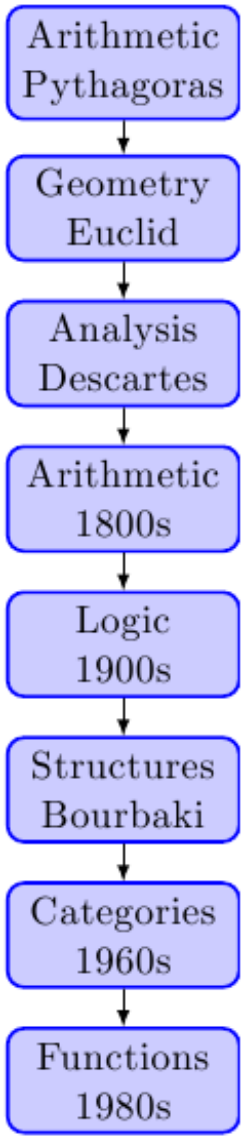
- Fields Medal
- Abel Prize
- Wolf Prize
- Millenium Prize

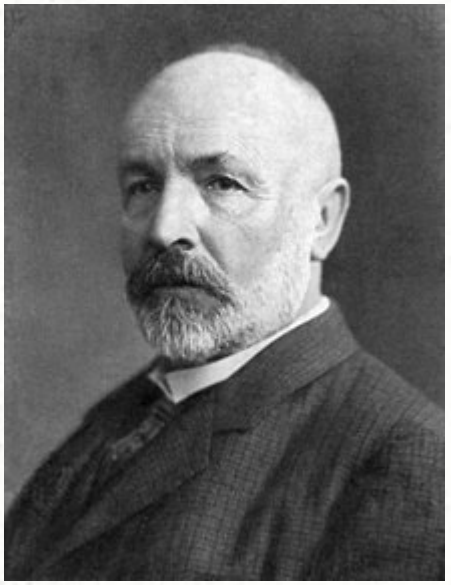




Evolution of Mathematics

- Pythagoreans - Arithmetic
- Euclid - Geometry
- Descartes – Analytic Geometry
- Newton – Calculus, Mechanics
- Euler – Numbers, Applications
- Gauss – Noneuclidean geometry
- Cantor – Set Theory, Infinity
- Frege – Predicate Logic
- Russell – *Principia Mathematica*
- Gödel – Incompleteness Theorems



Georg Cantor (1845-1918)



- Created set theory.

  How did Cantor Discover Set Theory and Topology?

  – Connection to Fourier series.

- Cardinality of sets.

- Not all infinities have same cardinality.

– Natural Numbers, Rationals, Reals

- Transfinite numbers and the Continuum hypothesis

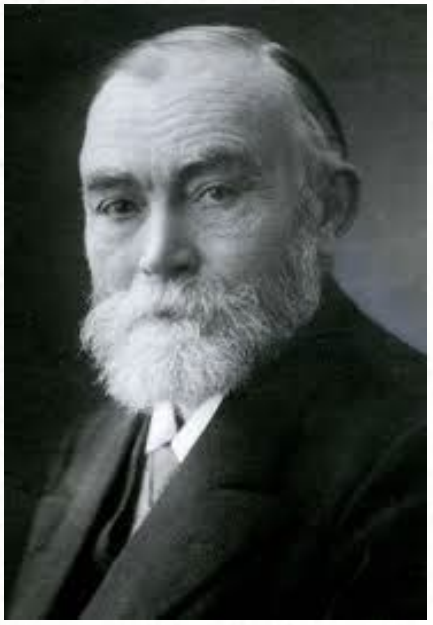
  There are no intermediate cardinal numbers between $\aleph_0$ (aleph-null) and the cardinality $\aleph_1$ of the continuum (set of real numbers).

Hilbert's Grand Hotel - 1924

- David Hilbert (1862-1923)
- Infinite # rooms fully occupied with infinite # people.
- First add one person.
- Then, a bus with an infinite number of guests.
- What about an infinite number of buses filled with an infinite number of people?

Gottlob Frege (1848-1925)

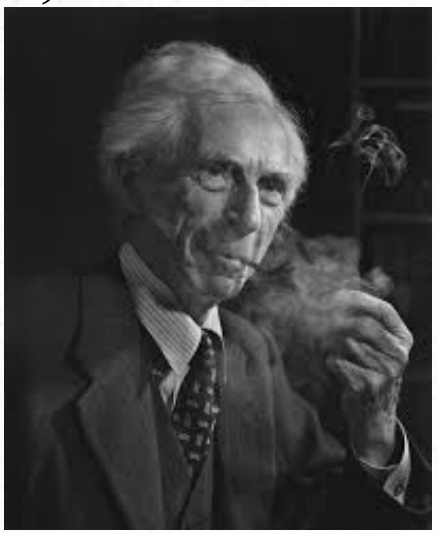
Invented axiomatic predicate logic,
Essential to



- Principia Mathematica (1910–13) Bertrand Russell, (1872–1970), and Alfred North Whitehead, (1861–1947).
- Kurt Gödel's (1906–78) incompleteness theorems.
- Alfred Tarski's (1901–83) theory of truth.
- Development of set theory.

Bertrand Russell, (1872–1970)

- Philosopher, logician, mathematician, historian, writer, essayist, social critic, political activist, and Nobel laureate



- Liar's Paradox

  "This sentence is a lie."

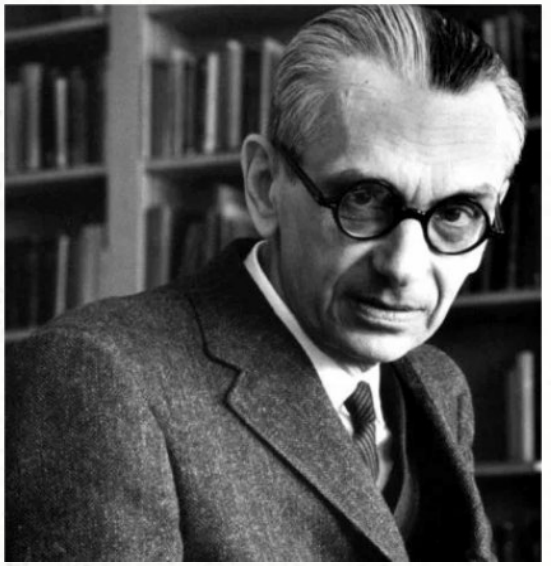
- Russell's Paradox

  Consider the set of all sets (power set) that are not members of themselves.

$$\text{Let } R = \{x \mid x \notin x\}, \text{ then } R \in R \iff R \notin R$$

- Avoid paradox - Zermelo–Fraenkel set theory

Kurt Gödel (1906–78)



- 1930 Incompleteness Theorems
- 1. If a logical, or axiomatic formal, system is consistent, it cannot be complete.
- 2. The consistency of axioms cannot be proved within their own system.

- Met Einstein 1933.
- Moved to Princeton 1940.
- 1949 Rotating universes and time travel.


Henri Cartan


André Weil


Jean Dieudonné


Claude Chevalley


René de Possel


Charles Ehresmann


Pierre Samuel

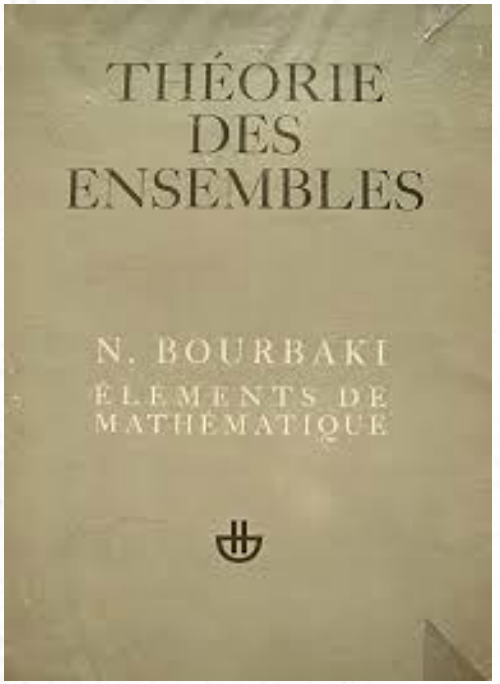

Jean-Pierre Serre


Laurent Schwartz


Adrien Douady

Nicolas Bourbaki (1935 -)

École Normale Supérieure, Paris



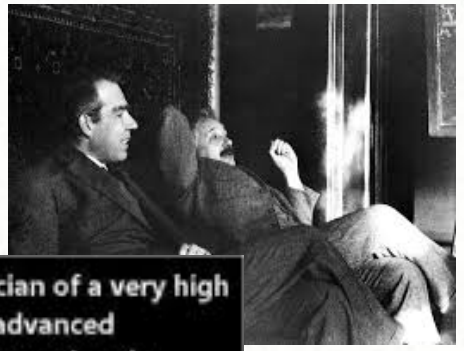
- Founders
  - Henri Cartan,
  - Claude Chevalley,
  - Jean Coulomb,
  - Jean Delsarte,
  - Jean Dieudonné,
  - Charles Ehresmann,
  - René de Possel, Szolem Mandelbrojt,
  - André Weil.
- Notable participants in later days:
  - Schwartz, Serre, Grothendieck, Eilenberg, and Lang.

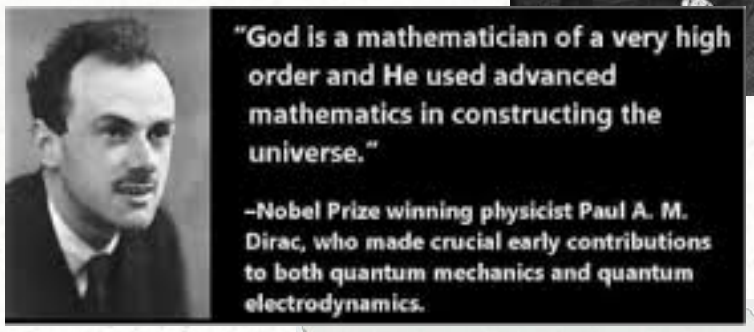
Bourbaki (1935 -)

École Normale Supérieure, Paris

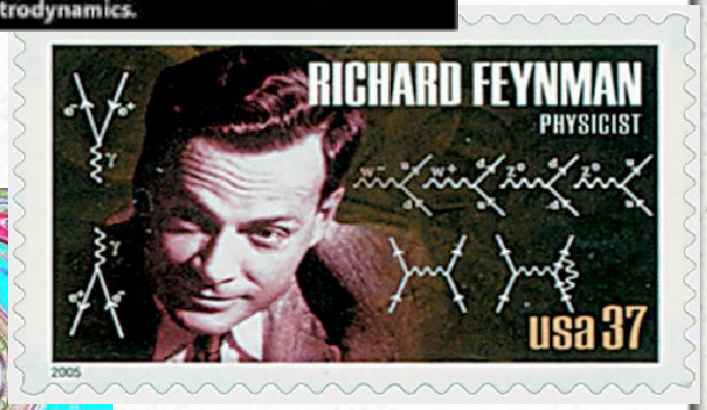
- Sought to write better analysis texts
  (Henri Cartan complained to Weil).
- Fall of French mathematics –due to WWI
  loss of a generation of French mathematicians.
- The rise of German Mathematics (and physicists).
- Bourbaki – a secret society, name based on past French general.
- Original plan -one volume of 1000 pages, collectively written, –
  a treatise on analysis. plan and became *Éléments de Mathématique*
- *By 1967:* 10 books in several volumes, over 60 chapters,
  publications starting in 1939. set Theory, Algebra, General Topology,
  Real Analysis, Topological Vector Spaces, Integration, Commutative
  Algebra, Varieties, Lie Groups and Algebras, Spectral Theory.
- In 2016 – Algebraic Topology; 2019 Revised Spectral Theory.

Revolutions – Paradigm Shifts

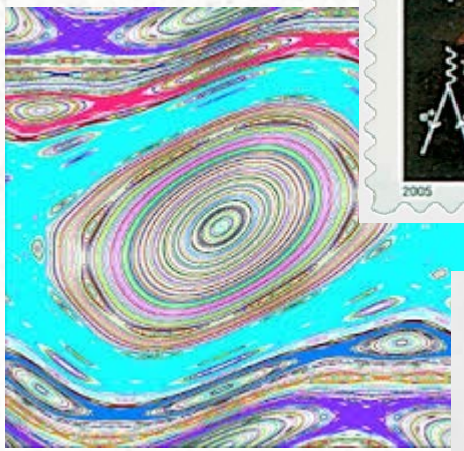




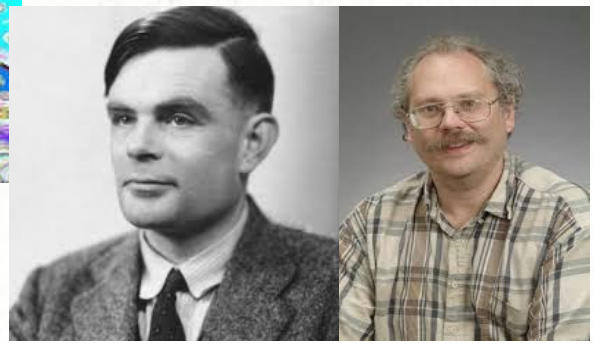
- Quantum Theory
- Special Relativity
- General Relativity
- String Theory Revolution



- Nonlinear Dynamics and Chaos



- Information Age



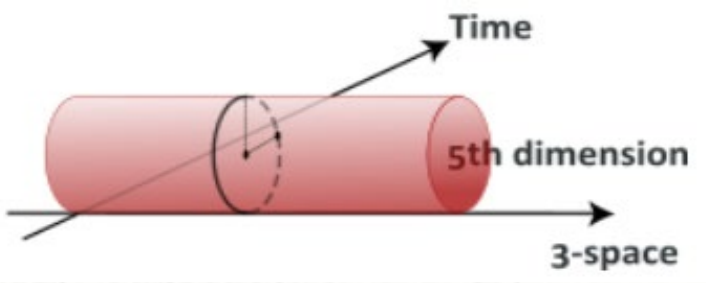
- *Focus on higher dimensions …*

Kaluza-Klein to Calabi-Yau

The Search for Higher Dimensions

- The 4th Dimension
  - Special Relativity 1905;
  - Minkowski – 4D spacetime.
  - General Relativity 1915.
- Theodor Kaluza (1885-1954) In 1921
  - Solved Einstein Equations in 5D.
  - Unified General Relativity with E&M.
- Oskar Klein (1894-1977) – QM Interpretation
  - Fifth dimension - curled up, microscopic
- Led to gauge theories on fiber bundles.

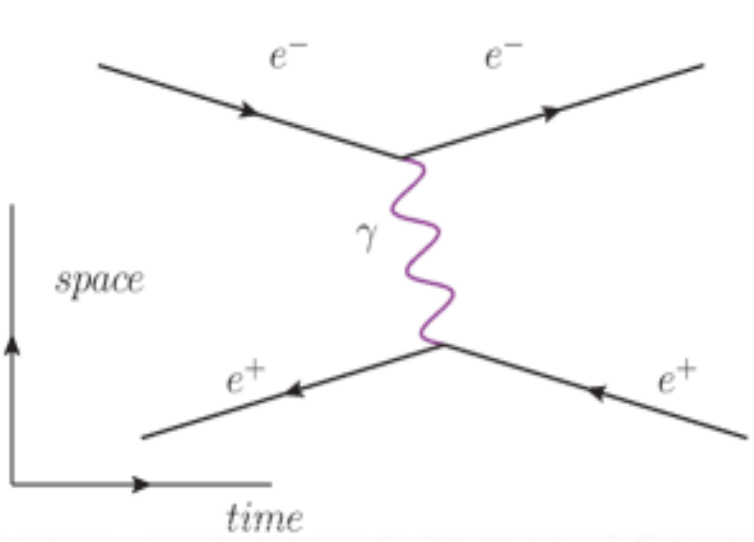

Theodor Kaluza




Oskar Klein

Feynman Diagrams and QED

- Richard Feynman (1918-1988)
- Quantum Electrodynamics (QED) – 1948-9
  - Interaction of light and matter
  - Nobel Prize with Schwinger, Tomonaga 1965

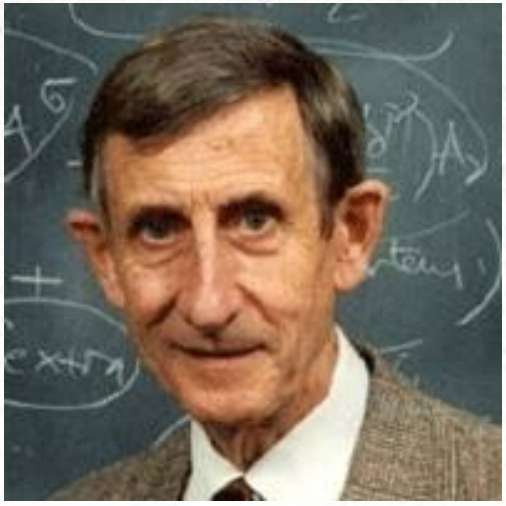


Not to F. Dyson

Freeman Dyson (1923-2020)

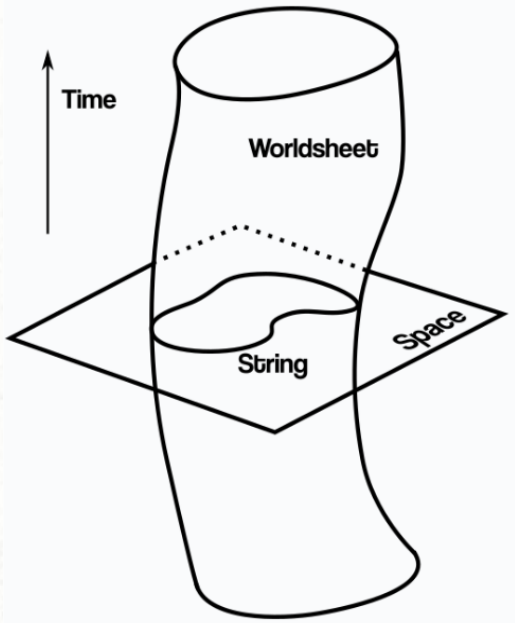
1972 Gibbs Lecture

I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has ended in divorce.



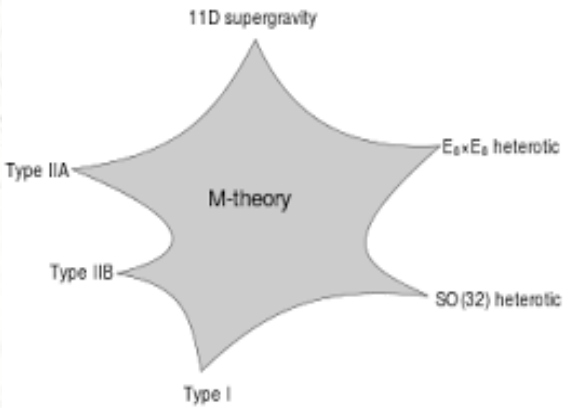
String Theory

- Gabriele Veneziano (1942-)
- In 1968 "Thumbing through old math books, they [Mahiko Suzuki] stumbled by chance on the **[Euler] Beta function** …," Michio Kaku



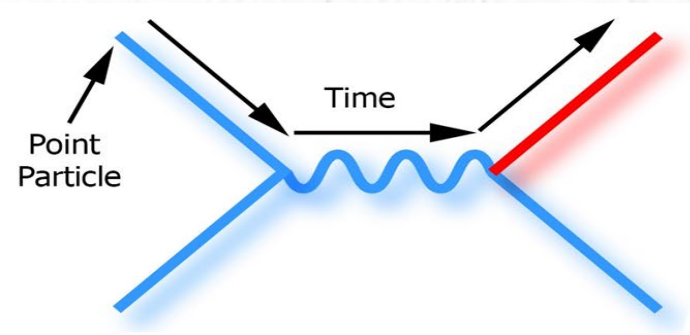
$$\mathrm{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$$

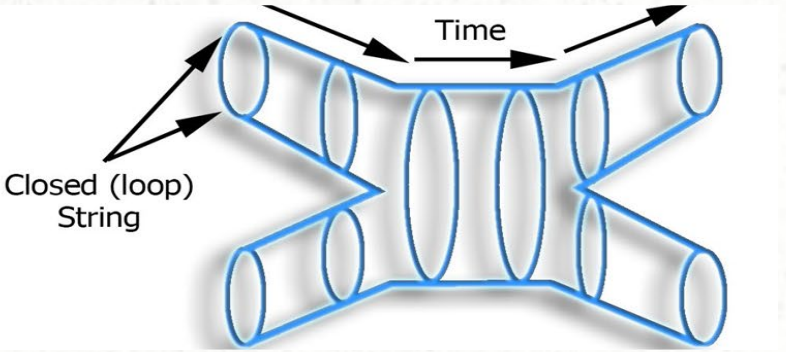
- Point-like particles modeled as 1D strings.
- Extra Dimensions 70s-90s
  - Bosonic string theory, 26-dimensional.
  - Superstring theory, 10-dimensional.
  - M-theory, 11-dimensional.



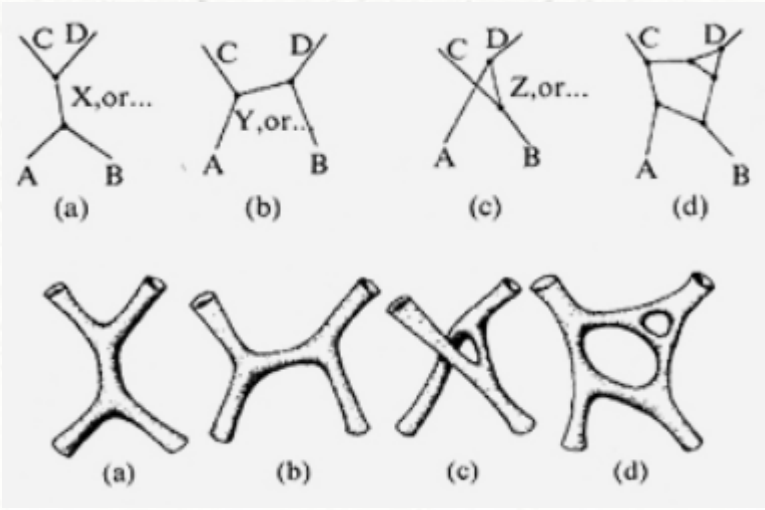
Feynman Diagrams to Strings

- Extend Feynman diagrams





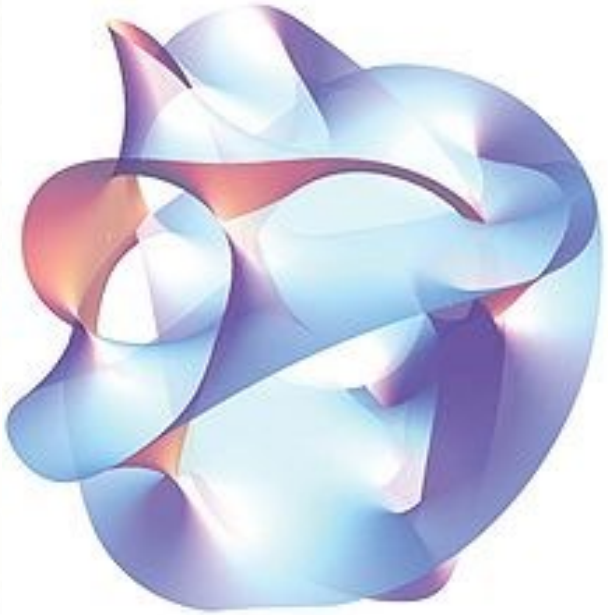
- Differential Geometry
- Topology
- Knot Theory



Calabi-Yau Manifolds

- Compactification (curl up extra 6 dimensions).
- Eugenio Calabi and Shing-Tung Yau, mathematicians

$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + \psi x_1 x_2 x_3 x_4 = 1$$



Mathematics and Physics

- Not Divorced

$\mathbb{R}^1 \times \left(\mathbb{R}^3 \backslash \{\vec{x}_1, \cdots \vec{x}_k\}\right) \supset \mathbb{C}P^1$ — spacetime around Yang-Mills monopoles

$\xrightarrow{c}$ nuclear force field sourced by monopole

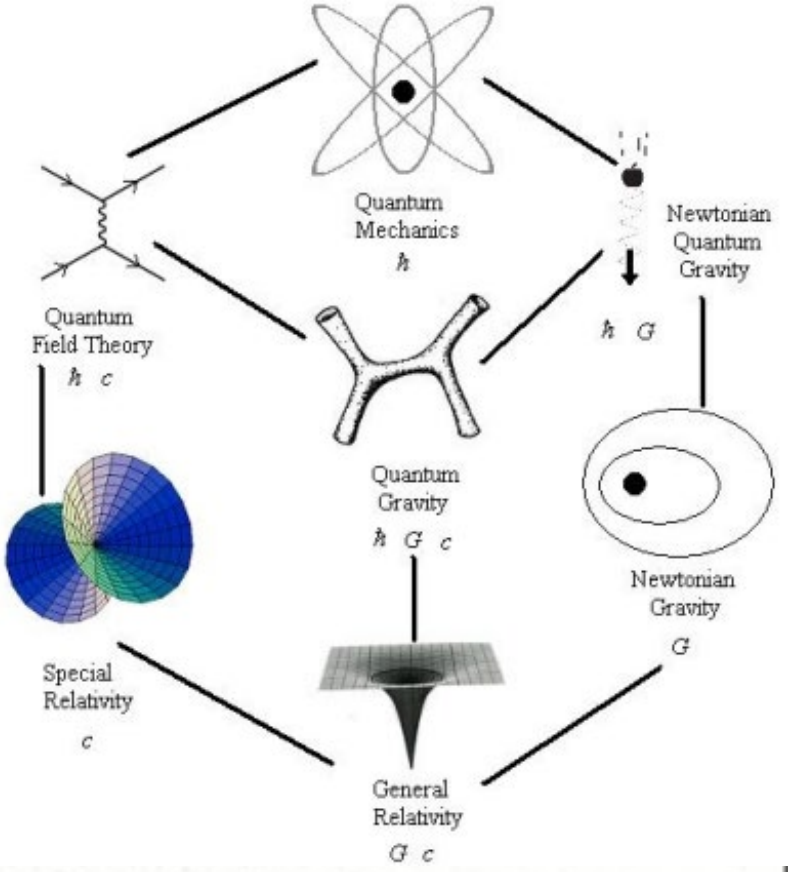
$\mathbb{C}P^1$ classifying space of complex Cohomotopy

$$[c] \in \left\{\mathbb{C}P^1 \longrightarrow \mathbb{C}P^1\right\}\Big/_{\sim_{\text{homotopy}}} \simeq \mathbb{Z}$$

charge lattice

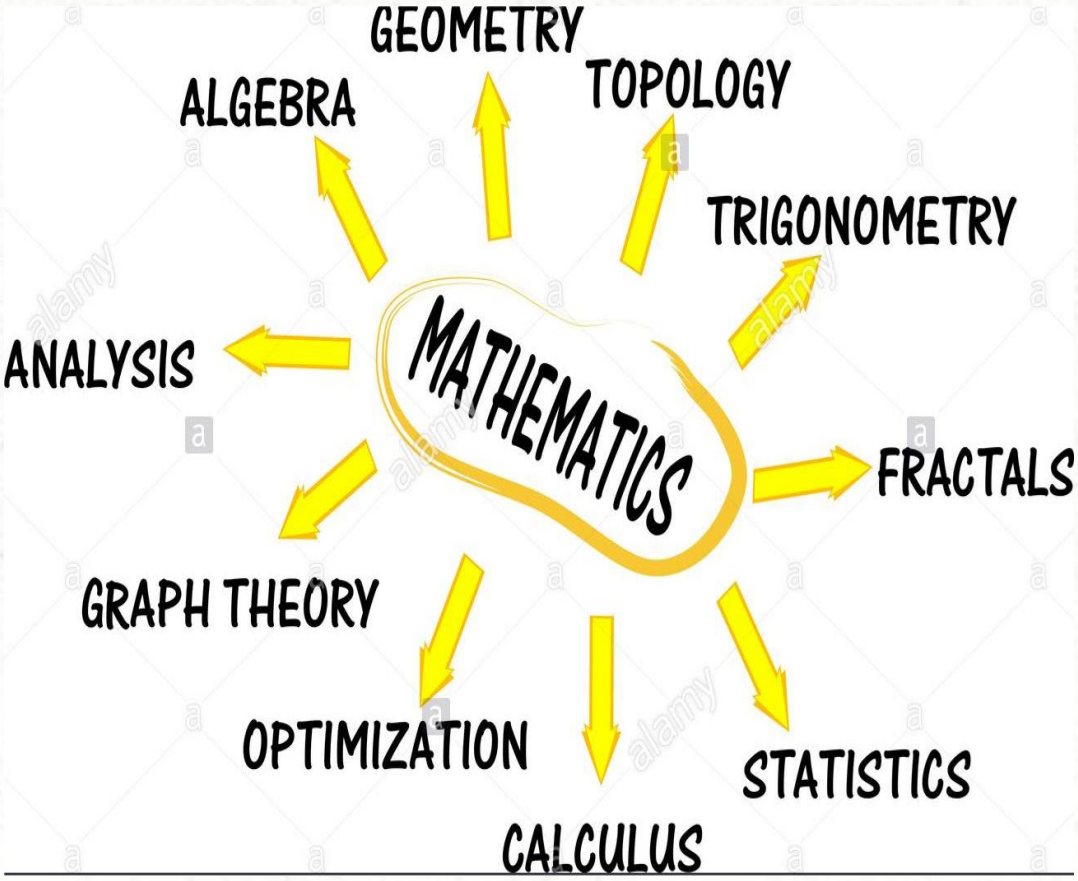
charge = homotopy class

Atiyah-Hitchin charge quantization – The moduli space of SU(2) Yang-Mills monopoles is the cocycle space of complex-rational Cohomotopy of any sphere enclosing them.



Michael Atiyah (1929-2019)

The Greatest Problems and Prizes



David Hilbert (1862-1943)

- Opened the International Congress of Mathematicians in Paris in the year 1900.
- Outlined 23 major mathematical problems to provide solutions for in the coming new century.

Hilbert's Problems 1-6

1. Cantor's problem of the cardinal number of the continuum. (partially resolved)
2. The compatibility of the arithmetic axioms.
3. The equality of two volumes of two tetrahedra of equal bases and equal altitudes. - (resolved)
4. Problem of the straight line as the shortest distance between two points. (vague)
5. Lie's concept of a continuous group of transformations without the assumption of the differentiability of the functions defining the group. (i.e., are continuous groups automatically differential groups?)
6. Mathematical treatment of the axioms of physics.

Hilbert's Problems 7-14

7. Irrationality and transcendence of certain numbers.
8. Problems (with the distribution) of prime numbers.
9. Proof of the most general law of reciprocity in any number field.

Red - Unresolved

10. Determination of the solvability of a diophantine equation.
11. Quadratic forms with any algebraic numerical coefficients.
12. Extension of Kronecker's theorem on abelian fields.
13. Impossibility of the solution of the general equation of the 7th degree.
14. Proof of the finiteness of certain complete systems of functions.

Hilbert's Problems 15-23

15. Rigorous foundation of Schubert's calculus.
16. Problem of the topology of algebraic curves and surfaces.
17. Expression of definite forms by squares.
18. Building space from congruent polyhedra.
19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?
20. The general problem of boundary curves.
21. Proof of the existence of linear differential equations having a prescribed monodromic group.
22. Uniformization of analytic relations by means of automorphic functions.
23. Further development of the methods of the calculus of variations.

Fields Medal

Prize (John Charles Fields)

- 2-4 mathematicians under 40 yrs.
- The International Congress of the International Mathematical Union
- Every four years.

Transire suum pectus mundoque potiri.
Rise above oneself and grasp the world.

See Fields Medalists

Abel Prize

- 1899 Proposed by the Norwegian mathematician Sophus Lie (1842-1899).
- He learned of Alfred Nobel's plans.
- First Awarded 2003
- See Laureates

Niels Henrik Abel (1802-1829)

Wolf Prize in Mathematics

- Awarded almost annually by the Wolf Foundation, in Israel.
- One of the 6 Wolf Prizes since 1978;
  - Agriculture, Chemistry, Medicine, Physics and Arts.
- See Winners

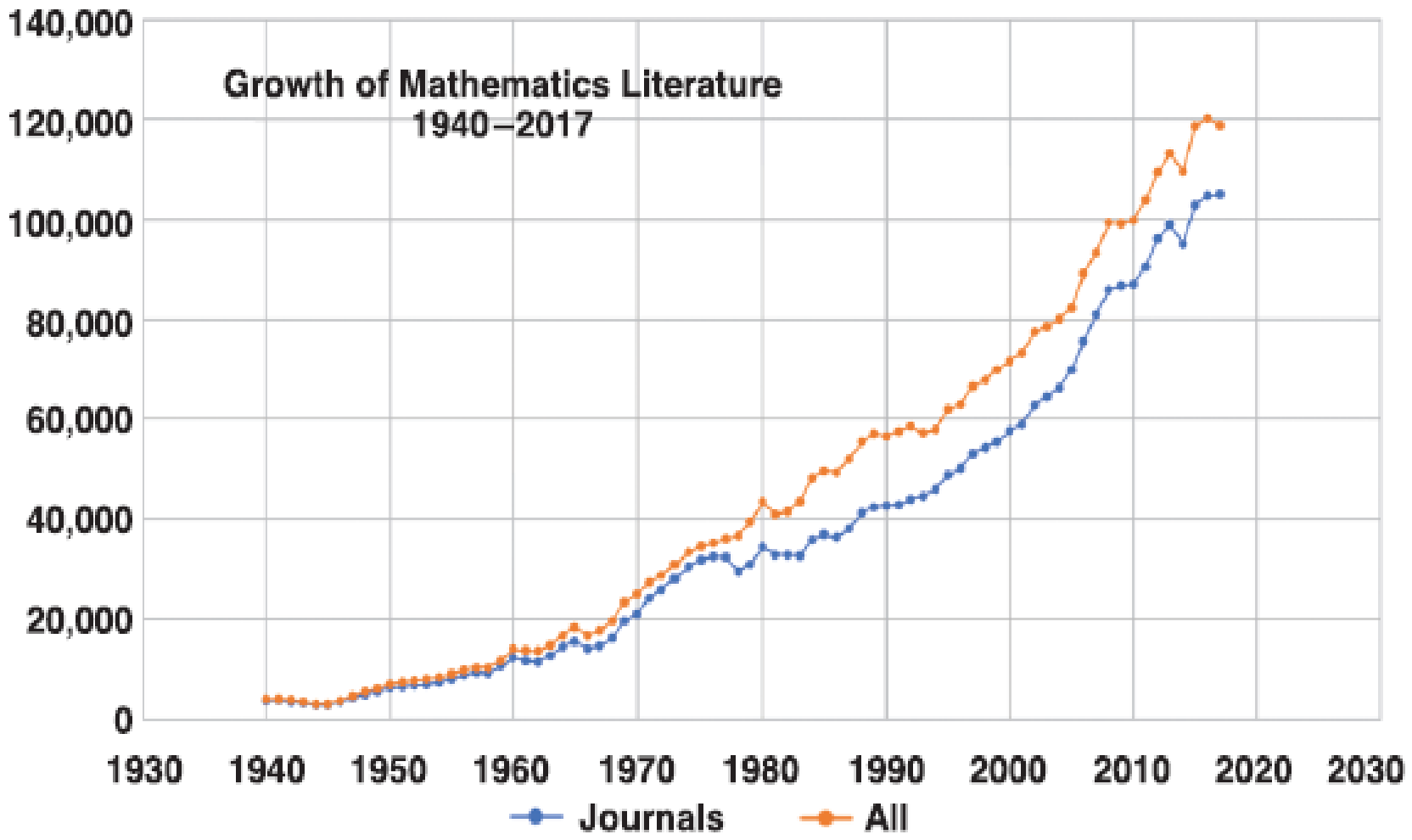
Greg Lawler 2019 	Charles Fefferman 2017 	George Mostow 2013 
Jean-François Le Gall 2019 	Richard Schoen 2017 	Michael Artin 2013 
Vladimir Drinfeld 2018 	James Arthur 2015 	Luis Caffarelli 2012 
Alexander Beilinson 2018 	Peter Sarnak 2014 	Michael Aschbacher 2012 

Millennium Prize Problems

Stated by the Clay Mathematics Institute on May 24, 2000 – for One Million Dollars

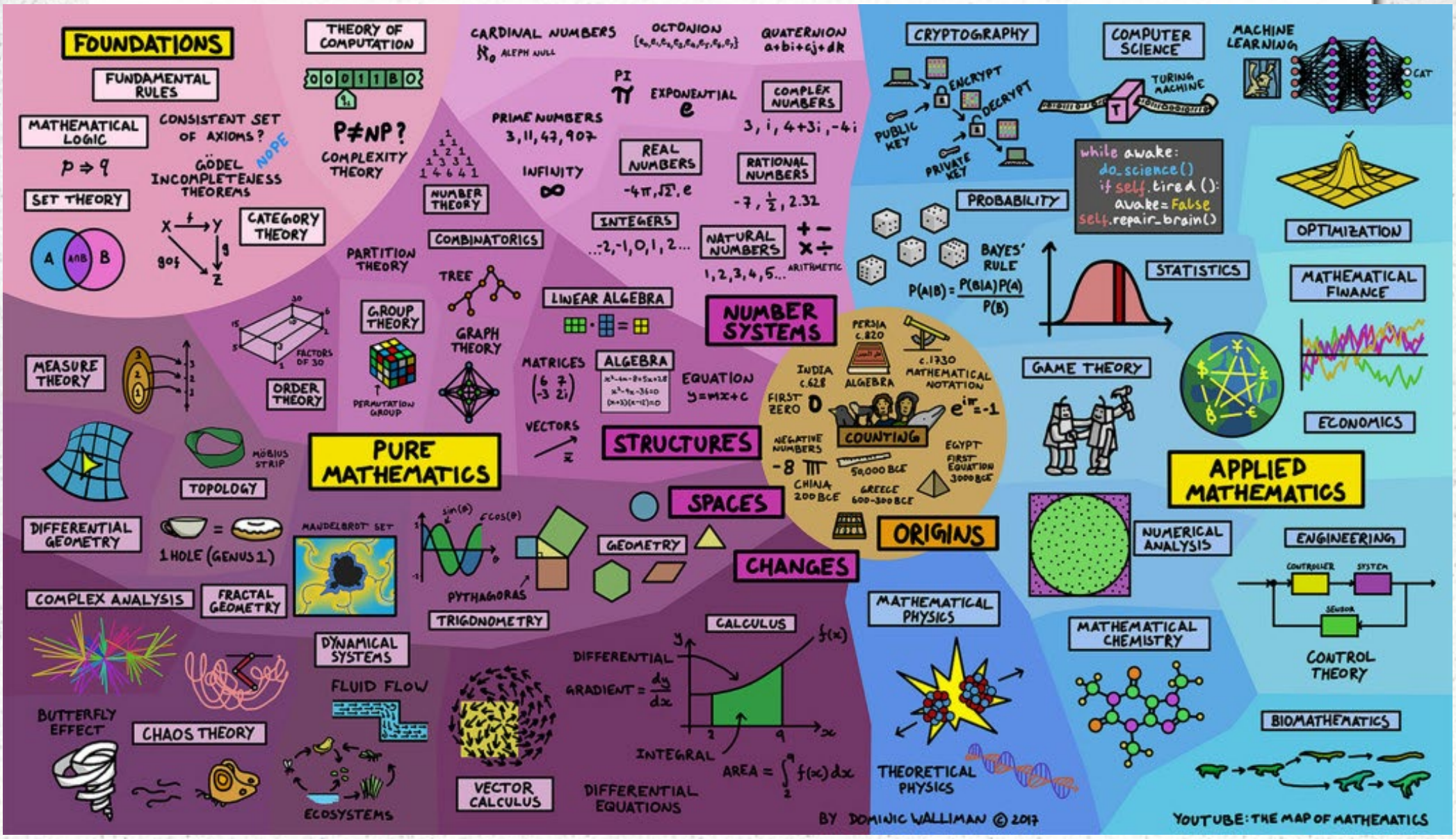
- Birch and Swinnerton-Dyer conjecture,
- Hodge conjecture,
- Navier–Stokes existence and smoothness,
- P versus NP problem,
- Poincaré conjecture (Solved! - Perelman),
- Riemann hypothesis, and
- Yang–Mills existence and mass gap

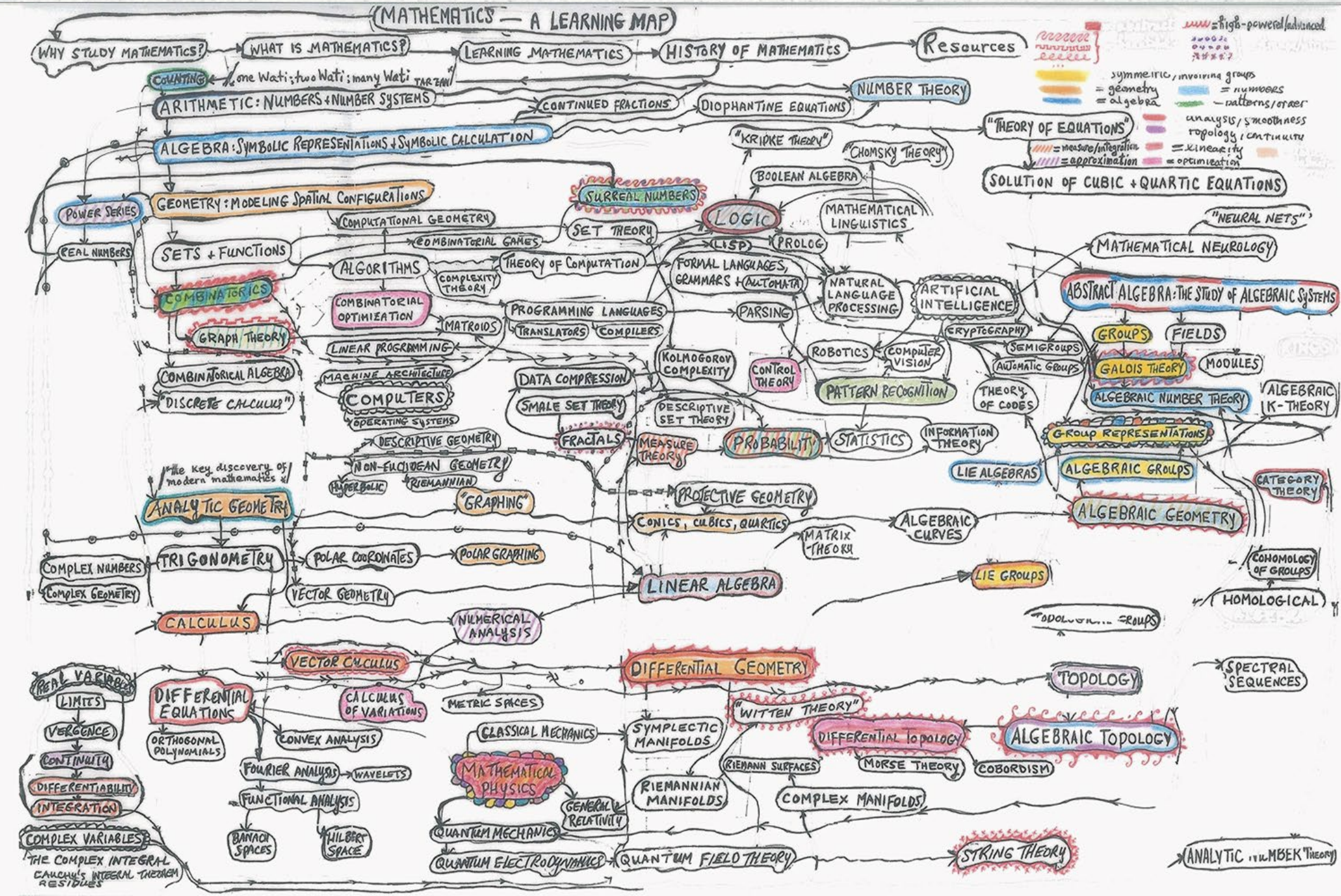
Mathematics Publications



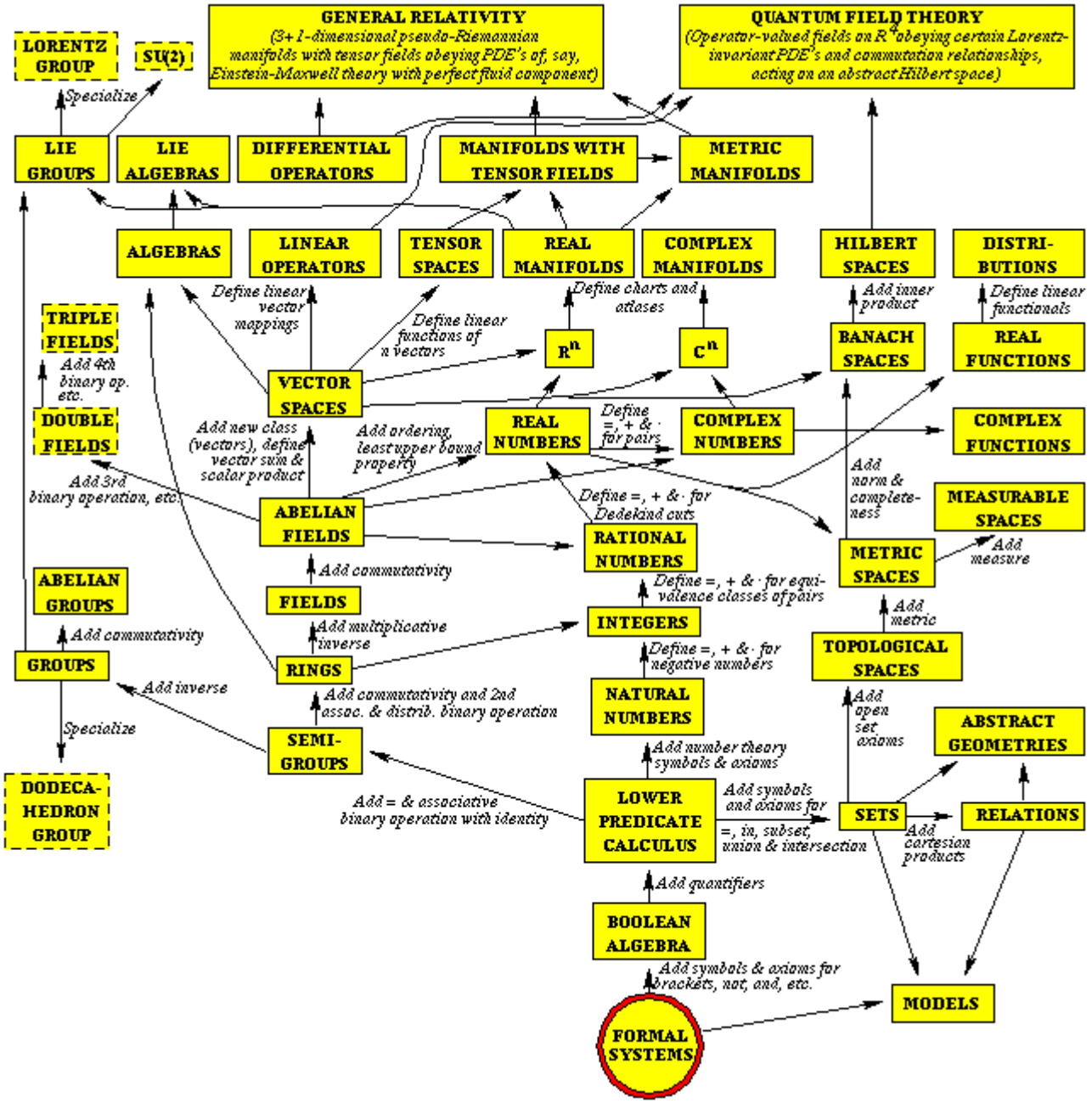
Mind Maps of Mathematics

https://www.sciencealert.com/this-mind-boggling-map-explains-how-everything-in-mathematics-is-connected-3





https://math.stackexchange.com/questions/166862/detailed-diagram-with-mathematical-fields-of-study


GENERAL RELATIVITY
(3+1-dimensional pseudo-Riemannian manifolds with tensor fields obeying PDE's of, say, Einstein-Maxwell theory with perfect fluid component)
QUANTUM FIELD THEORY
(Operator-valued fields on R^4 obeying certain Lorentz-invariant PDE's and commutation relationships, acting on an abstract Hilbert space)
LORENTZ GROUP
SU(2)
Specialize
LIE GROUPS
LIE ALGEBRAS
DIFFERENTIAL OPERATORS
MANIFOLDS WITH TENSOR FIELDS
METRIC MANIFOLDS
ALGEBRAS
LINEAR OPERATORS
TENSOR SPACES
REAL MANIFOLDS
COMPLEX MANIFOLDS
HILBERT SPACES
DISTRI-BUTIONS
Define linear vector mappings
Define linear functions of n vectors
Define charts and atlases
Add inner product
Define linear functionals
TRIPLE FIELDS
Add 4th binary op. etc.
DOUBLE FIELDS
VECTOR SPACES
R^n
C^n
BANACH SPACES
REAL FUNCTIONS
Add new class (vectors), define vector sum & scalar product
Add ordering, least upper bound property
REAL NUMBERS
Define =, + & · for pairs
COMPLEX NUMBERS
COMPLEX FUNCTIONS
Add 3rd binary operation, etc.
ABELIAN FIELDS
Define =, + & · for Dedekind cuts
Add norm & complete-ness
MEASURABLE SPACES
RATIONAL NUMBERS
METRIC SPACES
Add measure
ABELIAN GROUPS
Add commutativity
FIELDS
Define =, + & · for equi-valence classes of pairs
Add metric
INTEGERS
TOPOLOGICAL SPACES
Add multiplicative inverse
Define =, + & · for negative numbers
GROUPS
RINGS
Add inverse
NATURAL NUMBERS
Add open set axioms
ABSTRACT GEOMETRIES
Add commutativity and 2nd assoc. & distrib. binary operation
Specialize
Add number theory symbols & axioms
SEMI-GROUPS
DODECA-HEDRON GROUP
Add = & associative binary operation with identity
LOWER PREDICATE CALCULUS
Add symbols and axioms for =, in, subset, union & intersection
SETS
RELATIONS
Add cartesian products
Add quantifiers
BOOLEAN ALGEBRA
Add symbols & axioms for brackets, not, and, etc.
FORMAL SYSTEMS
MODELS

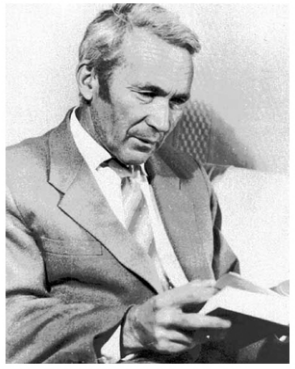
Top Numerical Algorithms (2000)

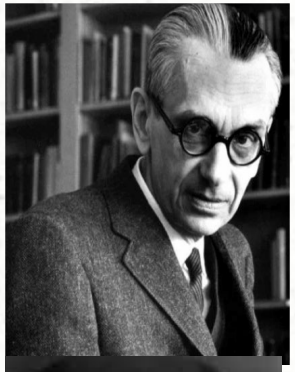
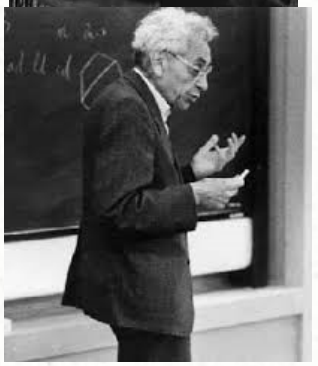
Another list (2015)

1. Newton and quasi-Newton methods
2. Matrix factorizations (LU, Cholesky, QR)
3. Singular value decomposition, QR and QZ algorithms
4. Monte-Carlo methods
5. Fast Fourier transform
6. Krylov subspace methods (conjugate gradients, Lanczos, GMRES, minres)
7. JPEG
8. PageRank
9. Simplex algorithm
10. Kalman filter

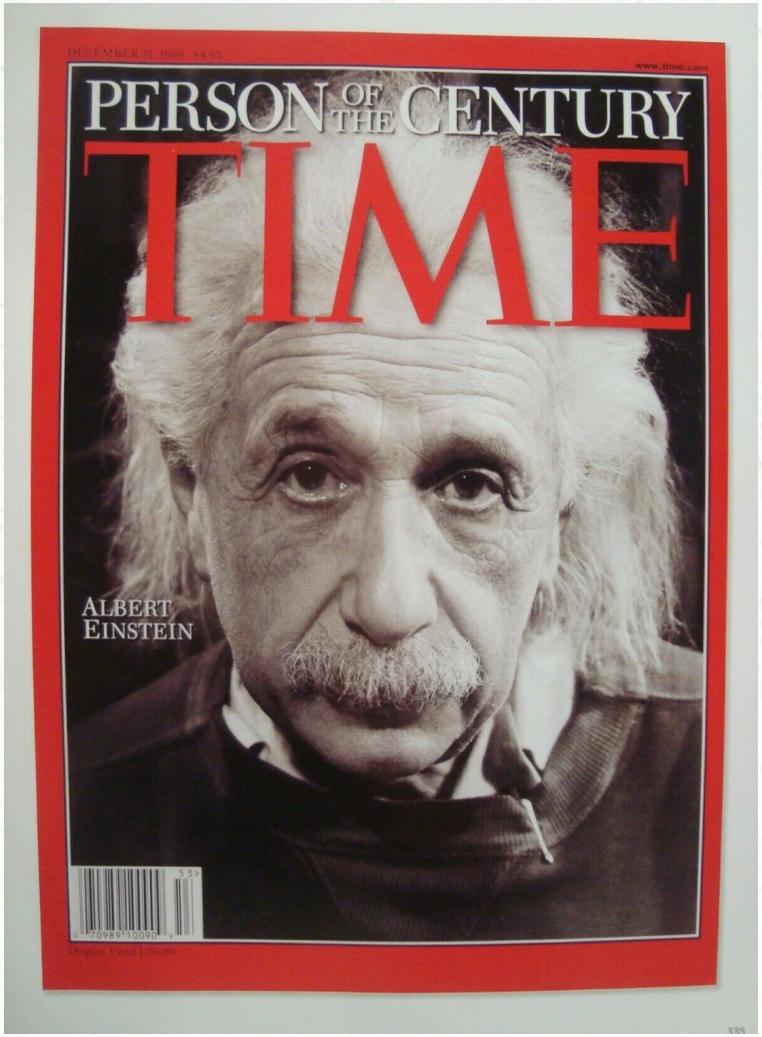
Greatest Mathematicians of 20th Century?

- John von Neumann
- Andrey Kolmogorov
- Claude Shannon
- Alexander Grothendieck
- Kurt Godel
- Paul Erdos
- Alan Turing
- Hermann Weyl
- Srinivasa Ramanujan


Person of the Century



Good Luck on Your Finals!