Introduction to the History of Mathematics

Fall 2019 - R. L. Herman
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Early Civilization

- Egypt - 3100 BCE
- Mesopotamia, or Babylonia - 2100 BCE
- China 1600 BCE
- India 1200 BCE

Arithmetic, Geometry, No proofs
Problems were practical or recreational

Figure 1: Babylonian tablet - Base 60
Greek Civilization

- Deductive Reasoning
  - Definitions, Axioms
  - Propositions via logic
- Geometry, Trigonometry, Astronomy, Numbers, Conics
- Pythagoras 6th Century BCE
- Euclid 4th Century BCE
  - Elements
- Archimedes 3rd Century BCE
- Appolonius 2nd Century BCE
- Thales 624-546 BCE,
  - Heron 10-70 CE
- Diophantus 200-284 CE,
  - Hypatia 400 CE

Figure 2: Euclid
Middle Eastern Mathematics - 700-1200 CE

In Europe - Dark Ages - 400-1200 CE

- Middle East - Islam 7th Century
- Translations of Greek Mathematics into Arabic
- Arabic Numerals by 1000 CE
- Algebra 825 CE - al-Khwarizmi - called al-Jabr
- Omar Khayyam (1048-1131) - geometric solution of cubic

Figure 3: al-Khwarizmi

Around 10th Century - Middle Eastern Mathematics brought to Spain.

It takes 300 years to accept Arabic numerals. - Fibonacci - 1202
Beyond Numerals

- Fractions 4000 years ago
- Sexagesimal (base 60) into 17th century
- Decimal (base 10)
  - al-Uqlidisi - (920-980)
  - al-Kashi (1380-1429)
  - Simon Stevin (1548-1620)
- Logarithms
  - John Napier (1550-1617) used a strange base
  - Henry Briggs (1561-1630) Base 10 Tables
  - 54 square roots of 10 leading to 30 decimal places
  - Tables to 14 decimal places

Figure 4: Brigg’s Tables
Italian Mathematics - Polynomial Equations, Complex Numbers

- Fibonacci (Leonardo of Pisa) (1170-1250) *Liber Abaci*
- Equation Solving contests
- Solutions of cubic and quartic
- Depressed cubic
del Ferro (1465-1526)
- Cubic and quartic equations
  Tartaglia (1500-1557)
  Cardano (1501-1576)
  *Ars Magna*
  Ferrari (1522-1565)
  Bombelli (1526-1572)
- Viète (1540-1603)
  Adriaan van Roomen
  Problem

*Figure 5: Cardano and Tartaglia*
Unification of Geometry and Algebra

- **Symbolic Algebra**
  - Rhetorical until 15th century
  - Syncopated/abbrev. - 1500
  - Symbolic algebra developed 16-17th century

- **Unification**
  - Oresme (1320-1382) - Velocity-time graphs, $\sum \frac{1}{n}$
  - Descartes (1596-1650)
    - Rep. curves by equations
    - Coordinate systems - published *The Method*
    - Made use of variables which can vary continuously - lines.
  - Fermat (1607-1665)
    - Rep. equations by curves

*Figure 6: Fermat and Descartes*
The Rise of Calculus

- Archimedes - 3rd century BCE
- Kepler (1571-1630)
- Cavalieri (1598-1647)
- Fermat (1607-1665)
- Wallis (1616-1673)
- Pascal (1623-1662)
- Barrow (1630-1677)
- Wren (1632-1723)
- Gregory (1638-1675)
- Newton (1642-1726)
  - *Principia* 1687
- Leibniz (1646-1716)
  - Notation $\frac{d}{dx}, \int$

Figure 7: Archimedes, Cavalieri, Wallis, Gregory, Newton, and Leibniz
The Infinitesimal

- Hippasus 500 BCE
  - Pythagorean
  - $\sqrt{2}$ irrational
- Introduction of Infinitesimals
  - Cavilieiri and Torricelli
  - Stevin, Wallis, Harriot
- Critics
  - Jesuits in Italy
  - George Berkeley (1685-1753)
    - *The Analyst, - A Discourse Addressed to an Infidel Mathematician*, 1734
  - infinitesimals undermine mathematics and rationality
- Augustin-Louis Cauchy - 1821

**Figure 8:** Berkeley’s *The Analyst*
Exploiting Calculus

- Bernoulli Family
- Euler (1707-1783)
- Laplace (1749-1827)
- Neptune discovered using math - 1846

Figure 9: Euler and Laplace
The Birth of Rigor - 19th Century

Non-Euclidean Geometry

- Parallel Postulate
- Hyperbolic Geometry
  - Nikolai Lobachevsky (1792-1856)
  - Johann Bolyai (1802-1860)
  - Johann Carl Friedrich Gauss (1777-1855)
- Elliptic Geometry
  - Georg Friedrich Bernhard Riemann (1826-1866)
  - Prince of Mathematicians
- By 1870’s doubted Euclid

Figure 10: Gauss, Lobachevsky, Bolyai

Figure 11: Different Geometries
19th Century Group Theory

- Joseph-Louis Lagrange (1736-1813)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Niels Henrik Abel (1802-1829)
- Évariste Galois (1811-1832)
- Arthur Cayley (1821-1895)
- Camille Jordan (1838-1922)

Figure 12: Abel and Galois
19th Century Analysis and Set Theory

- Jean-Baptiste Joseph Fourier (1768-1830)
- Johann Carl Friedrich Gauss (1777-1855)
- Augustin Cauchy (1789-1857)
- Karl Weierstrass (1815-1897)
- George Boole (1815-1864)
- Georg Friedrich Bernhard Riemann (1826-1866)
- Richard Dedekind (1831-1916)
- Georg Ferdinand Ludwig Philipp Cantor (1845-1918)
  - Founder of set theory
  - Defined infinite sets

Figure 13: Gauss and Riemann
19th Century Number Theory

- Marie-Sophie Germain (1776-1831)
- Johann Carl Friedrich Gauss (1777-1855) *Disquisitiones Arithmeticae* - 1801
- Adrien-Marie Legendre (1752-1833) and Peter Gustav Lejeune Dirichlet (1805-1859) prove Fermat’s Last Theorem for $n = 5$ in 1825
  - Dirichlet, $n = 14$ in 1832.
- Riemann hypothesis, distribution of primes - 1832.
- Charles Jean de la Vallé-Poussin and Jacques Hadamard - Prime Number Theorem. 1896

*Figure 14: Sophie Germain, Adrien-Marie Legendre*
The Modern Era

Figure 15: Hilbert, Gödel, Uhlenbeck, Ramanujan, Wiles, Mirzakhani, Shannon, Russell, Noether

- Chronology of 20th Century Mathematicians
- Greatest Mathematicians born between 1860 and 1975
- Pictures of Famous 20th Century Mathematicians
- The Story of Math