1. Terminology

- · adaptive integration
- · bisection method
- bracketing
- · Broyden's update
- · cardinal functions
- cubic spline
- discretization
- · extrapolation
- finite difference
- fixed point iteration
- · fixed point problem
- · Gauss-Newton method
- · golden ratio
- · hat functions
- interpolation
- interpolant
- inverse interpolation
- · Jacobian matrix
- · Levenberg's method
- · linear convergence
- misfit function
- · Newton's method
- Newton-Cotes formula
- · nonlinear least squares problem
- Nodes
- · order of accuracy
- piecewise linear interpolation
- · piecewise polynomial
- · Quasi-Newton methods
- quadratic convergence
- quadrature
- residual
- · rootfinding problem
- · secant method
- simple root
- Simpson's rule
- steepest descent
- superlinear convergence
- · trapezoid formula
- truncation error

· weights - Fornberg and quadrature

2. Algorithms

- Bisection
- Fixed point iteration
- · Newton's method
- · Secant method
- Multidimensional Newton Iteration, $J(x_k)\mathbf{s}_k = -\mathbf{f}(\mathbf{x}_k)$
- Quasi-Newton Methods,

- Finite difference
$$J(\mathbf{x}) \approx \frac{\mathbf{f}(\mathbf{x} + \delta \mathbf{e}_j) - \mathbf{f}(\mathbf{x})}{\delta}$$

- Broyden's update, $A_k \approx J(\mathbf{x}_k)$, $A_{k+1} = A_k + \frac{\mathbf{f}_{k+1} - \mathbf{f}_k - A_k \mathbf{s}}{\mathbf{s}_k^T \mathbf{s}_k} \mathbf{s}_k^T$

Linear interpolation

$$p(x) = y_k + \frac{y_{k+1} - y_k}{t_{k+1} - t_k} (x - t_k), x \in [t_k, t_{k+1}]$$

- Hat function interpolant $p(x) = \sum_{k=0}^{n} c_k H_k(x)$.
- · Forward and backward finite differences
- · Centered finite differences
- · Fornberg's algorithm
- Trapezoidal formula

3. Be able to

- Apply conditioning to rootfinding problems.
- · Create a cobweb diagram.
- Predict convergence to fixed point.
- Apply Newton's and secant methods.
- Describe approaches to solving nonlinear systems.
- Explain the differences between square and overdetermined linear/nonlinear systems.
- Describe role of λ in Levenberg method, $(A_k^T A_k + \lambda I) \mathbf{s}_k = -A_k^T \mathbf{f}_k$.
- Explain role of misfit function in nonlinear data fitting.
- Interpolate small data set.
- Explain conditions leading to cubic splines.
- Compute finite difference approximations and truncation errors for derivatives.
- Find a Newton-Cotes quadrature formula.
- · Relate Trapezoid to Simpson methods.