MAT 261 Exam IV

Name ______________________

Problem 1.

a. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Compute

i. Ñ · **r** =

ii. Ñ × **r** =

b. Complete the identity:

$\nabla \cdot (f\,\nabla g) =$

c. Sketch the vector field $\mathbf{F} = \dfrac{y\mathbf{i} - x\mathbf{j}}{\sqrt{x^2 + y^2}}$. Use enough vectors in the plane to show the general behavior.

d. Is the vector field $\mathbf{F} = e^y\mathbf{i} + xe^y\mathbf{j}$ conservative? ___
Explain your answer.

Instructions:

a. Do all of your work in this booklet. Do not tear off any sheets.
b. **Show all of your steps** in the problems for full credit.
c. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
d. Place a **box around your answers**.
e. Place your **name on all of the pages**.
f. If you need more space, you may use the back of a page and write **On back of page #** in the problem space**.**

DO AS MANY PROBLEMS AS YOU CAN!

Column	**Pts**
1 (12 pts)	
2 (13 pts)	
3 (14 pts)	
4 (11 pts)	
Total (50 pts)	

Problem 2.

a. Finish each expression

i. Fundamental Theorem for Line Integrals.

$\int_C$ Ñ$f \cdot d\mathbf{r} =$

ii. Green's Theorem in the Plane

$\int_C P\,dx + Q\,dy =$

b. Which of the following expressions are meaningful? ______________

A. curl f B. div (grad f) C. grad(curl **F**)
D. div(div **F**) E. curl(grad f) F. div(curl(grad f))

c. Compute $\int_C y\,dx - x\,dy$ for the path C given by $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$.

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Problem 3.

a. Use Green's Theorem to evaluate $\oint_C xy\,dy - y^2\,dx$ for C the unit square traversed with a positive orientation.

b. Find the area of the surface which is the part of the plane $x + 2y + 3z = 12$ that lies inside the cylinder $x^2 + y^2 = 9$.

c. Consider the surface: $\mathbf{r}(u,v) = < u^2, u\sin v, u\cos v >$, $0 \le u \le 1$, $0 \le v \le 2\pi$.

i. Find a normal to the surface at $(u,v) = (1,0)$.

ii. What is the equation of the tangent plane at this point?

iii. Set up the integral over u and v for the area of this surface. **Do not integrate.**

Problem 4.

a. Let $\mathbf{F}(x,y) = < 3 + 2xy, x^2 - y^2 >$. Find a function $f(x,y)$ such that Ñ$f = \mathbf{F}$ for all (x,y).

b. Consider the vector field $\mathbf{F} = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$. Let S be the surface of the standard unit cube.

i. Write the integral for the flux of $\mathbf{F}$ across S

iii. Use Gauss' Theorem to rewrite this flux as a volume integral over the cube. **Do not integrate.**

c. Use Stoke's Theorem to compute the work done by the force field $\mathbf{F} = < x - y^2, y - z^2, z - x^2 >$ when a particle moves around the triangle with vertices (1,0,0), (0,1,0), and (0,0,1). [Note: The triangle lives in the plane $x + y + z = 1$.]