

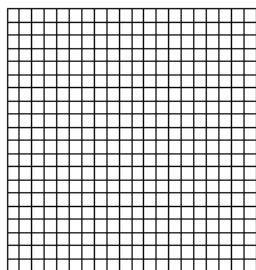
MAT 261 Exam II

Name _____

Problem	Score
1 (15 pts)	
2 (16 pts)	
3 (17 pts)	
4 (12 pts)	
Total (60 pts)	

1. (15 pts) Let $f(x, y) = x + y^2 + 1$ for the following:

- a. Compute ∇f .
- b. Find $\frac{\partial^2 f}{\partial y^2}$.
- c. Find the directional derivative at (1,1) in the direction $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.
- d. Sketch and label three level curves for this function.



e. Express df in terms of the differentials dx and dy .

2. (16 pts) Consider the curve $\mathbf{r}(t) = \langle 3 \sin 2t, -1, 3 \cos 2t \rangle$.

- a. Find $\mathbf{r}'(t)$.
- b. Find $\mathbf{r}''(t)$.
- c. Find $|\mathbf{r}'(t)|$.
- d. Find the length of the curve from $t = 0$ to $t = \frac{\pi}{4}$.
- e. Find the normal to the curve, $\mathbf{N}(t)$, at $t = \pi$.
- f. Determine the curvature at $t = \pi$.

Bonus What is the centripetal acceleration, a_N , in Problem 2 at $t = \pi$?

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3. (17 pts) Do the following:

a. Find $\frac{\partial z}{\partial y}$ when $yz = \sin(xz)$.

b. Let $z = xy^2 - 3x^3$, $x = s^2 + t^2$, $y = s - 2t$. Find $\frac{\partial z}{\partial s}$ when $s = 0$ and $t = 1$.

c. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2y^2}{x^2 + y^2}$ does not exist.

d. A particle starts at the origin with initial velocity $\mathbf{v}(0) = 2\mathbf{k}$. If its acceleration is $\mathbf{a} = \langle 1, t, 0 \rangle$, then what is the particle's position at $t = 1.0$.

e. A cylindrical can is measured to have a height of 11.5 ± 0.1 cm and a radius of 2.2 ± 0.2 cm. Determine the volume and the error, ΔV , in computing the volume.

4. (12 pts) And finally ...

a. Determine the nature of the critical point(s) for $f(x, y) = x^2 + xy + y^2$.

b. Find the maximum and minimum values of $f(x, y) = 2x + y$ subject to the constraint $x^2 - y^2 = 1$.

c. Consider the surface $z = x^2 + xy + y^2$. At the point $P = (1, 1, 3)$ find

i. the normal to the surface at P, and

ii. the equation of the tangent plane at P.