Chapter 16 Review

1. Vector Fields
   a. Gradient Vector Fields $\mathbf{F} = \nabla f$
   b. Conservative Vector Fields

2. Line Integrals
   a. Line Integrals $\int_C f(x,y)\,ds = \int_a^b f(x,y)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
   b. Line Integral with respect to arc length $\int_C P(x,y)\,dx + Q(x,y)\,dy$
   c. Line Integrals in space
   d. Line Integrals of vector fields
      $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)dt = \int_C Pdx + Qdy + Rdz$
   e. Work

3. The Fundamental Theorem for Line Integrals
   a. Fundamental Theorem of Calculus: $\int_a^b F'(x)\,dx = F(b) - F(a)$
   b. Fundamental Thm for line Integrals: $\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$
   c. Path Independence
   d. Conservative Vector Fields in plane: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
   e. Conservation of Energy

4. Green'sTheorem
   a. $\int_C Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$
   b. Surface area $A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - y\,dx$

5. Curl and Divergence
   a. $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$
   b. grad $f = \nabla f$, curl $\mathbf{F} = \nabla \times \mathbf{F}$, div $\mathbf{F} = \nabla\cdot\mathbf{F}$, Laplace Operator: $\nabla^2 f$
   c. Identities
      i. $\text{curl}(\nabla f) = 0$ - relation to conservative fields
      ii. $\text{div}\,\text{curl}\,\mathbf{F} = 0$

d. Green's Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_D (\operatorname{curl}\mathbf{F})\bullet\mathbf{k}\, dA,\ \oint_C \mathbf{F}\bullet\mathbf{n}\, ds = \iint_D \operatorname{div}\mathbf{F}\, dA$

6. Parametric Surfaces and Their Areas
   a. Parameterization $x = x(u,v), y = y(u,v), z = z(u,v)$
   b. Planes - $\mathbf{r} = \mathbf{r}_0 + u\mathbf{a} + v\mathbf{b}$
   c. Surface of Revolution about x-axis: $\mathbf{r} =< x, f(x)\cos q, f(x)\sin q >$
   d. Tangent Plane: Normal Vector = $\mathbf{r}_u \times \mathbf{r}_v$
   e. Surface Area
      i. Surfaces: $A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v|\, dA,$
      ii. Graphs: $A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

7. Surface Integrals and Flux
   a. Surfaces: $\boxed{\iint_S f(x,y,z)\, dS = \iint_D f(\mathbf{r}(u,v))\, |\mathbf{r}_u \times \mathbf{r}_v|\, dA,}$
   b. Graphs: $\iint_S f(x,y,z)\, dS = \iint_D f(x,y,g(x,y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA,$
   c. Vector Fields $\boxed{\iint_S \mathbf{F}\bullet d\mathbf{S} = \iint_S \mathbf{F}\bullet\mathbf{n}\, dS}$
      i. Surfaces: $\iint_S \mathbf{F}\bullet d\mathbf{S} = \iint_D \mathbf{F}\bullet(\mathbf{r}_u \times \mathbf{r}_v)\, dA$
      ii. Graphs: $\iint_S \mathbf{F}\bullet d\mathbf{S} = \iint_D \left(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R\right) dA$

8. Stoke's Theorem: $\boxed{\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_D (\operatorname{curl}\mathbf{F})\bullet d\mathbf{S}}$

9. The Divergence Theorem: $\boxed{\oiint_S \mathbf{F}\bullet d\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\, dV}$