Chapter 15 Review

1. Double Integrals Over Rectangles
   a. Areas of Planar Regions $A(D) = \iint_D dA$
   b. Average Values $f_{ave} = \dfrac{1}{A(D)} \iint_D f(x,y)\, dA$
2. Iterated Integrals, $\iint_D f(x,y) dA = \int_c^d \int_a^b f(x,y)\, dxdy$
   a. Integrals over Rectangles
   b. Fubini's Theorem for Double and Triple Integrals
3. Double Integrals Over General Regions
   a. Setting Up Integrals Over General Regions
   b. Change of Order of Integration
4. Polar Coordinates
   a. $x = r\cos\theta,\ y = r\sin\theta,\ r = \sqrt{x^2+y^2},\ \theta = \tan^{-1}\dfrac{y}{x}$
   b. Area $dA = r\, drd\theta$
5. Applications of Double Integrals
   a. Center of Mass $x_{cm} = \dfrac{1}{M}\iint_D x\rho\, dA,\quad y_{cm} = \dfrac{1}{M}\iint_D y\rho\, dA,\ M = \iint_D \rho\, dA$
   b. Moments of Inertia
   $I_x = \dfrac{1}{M}\iint_D y^2\rho\, dA,\quad I_y = \dfrac{1}{M}\iint_D x^2\rho\, dA,\quad I_o = \dfrac{1}{M}\iint_D \left(x^2+y^2\right)\rho\, dA$
6. Surface Area for $z = f(x,y)$: $\quad A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$
7. Triple Integrals
   a. Over Boxes $\iiint_B f(x,y,z)\, dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\, dxdydz$
   b. Applications, $M = \iiint_E \rho(x,y,z)\, dV$, etc.
8. Cylindrical and Spherical Coordinates
   a. Cylindrical Coordinates $x = r\cos\theta, y = r\sin\theta, z = z$
   b. Spherical Coordinates $x = \rho\cos\theta\sin\phi, y = \rho\sin\theta\sin\phi, z = \rho\cos\phi$
   c. Triple Integrals: $\iiint_E f(x,y,z)\, dV = \int_r^s \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta, z)\, rdrd\theta dz$
   $\iiint_E f(x,y,z)\, dV = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} g(\rho,\theta,\phi)\, \rho^2 \sin\phi\, d\rho d\theta d\phi$
9. Change of Variables in Multiple Integrals
   a. Transformation of Regions
   b. Computation of Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} \\ \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} \end{vmatrix}$ and $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$
   c. Change of Variables $\iint_R f(x,y) dA = \iint_S f(x(x,v), y(u,v)) \left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| dudv$