Chapter 14 Review

1. Functions of Several Variables
    a. Domains and Ranges, dependent and independent variables
    b. Level Curves $f(x, y) = k.$
    c. Level Surfaces $f(x, y, z) = k$
2. Limits and Continuity
    a. Existence and Computation of Limits
    b. Continuity
3. Partial Derivatives $f_x = \frac{\partial f}{\partial x}$
    a. Computation of First Order Partial Derivatives
    b. Implicit Differentiation
    c. Higher Order Derivatives f_{xx}, f_{xy}, f_{xxyz}, etc $f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$
    d. Clairaut's Theorem
    e. Harmonic Functions $f_{xx} + f_{yy} = 0$
4. Tangent Planes and Linear Approximation
    a. Equation of Tangent Plane Using Partial Derivatives
       $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$
    b. Linear Approximation (Linearization)
       $f(x, y) \approx f(a,b) + f_x(a,b)(x - a) + f_y(a,b)(y - b)$
    c. Increments $\Delta z = f_x(a,b)\Delta x + f_y(a,b)\Delta y$
    d. Differentials $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$
    e. Application of Linear Approximation, such as errors
5. The Chain Rule
    a. Chain Rule for One or Two Independent Variables $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$
    b. General Chain Rule
    c. Tree Diagrams
    d. Implicit Differentiation $\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$, etc
6. Directional Derivatives and the Gradient Vector
    a. Directional Derivative $D_{\mathbf{u}} f\big|_{(x_0, y_0)} = \nabla f \cdot \mathbf{u}\big|_{(x_0, y_0)}$
    b. Gradient Vector $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$
    c. Maximum and Minimum Rate of Change
        i. Direction of Extreme change and value

7. Maximum and Minimum Values
    a. Determining Critical Points $\nabla f = 0$
    b. Second Derivative Test $D(a,b) = \left[f_{xx} f_{xx} - f_{xy}^{2} \right]_{(a,b)}$
    c. Absolute Maxima and Minima – Extreme Value Theorem
8. Lagrange Multipliers
    a. Find Extrema of Functions with One Constraint $\nabla f = \lambda \nabla g$