

Chapter 12 Review

1. Three-Dimensional Coordinate Systems

- Coordinate Planes
- Distance Formula
- Equation of Sphere $x^2 + y^2 + z^2 + Ax + By + Cz + D = 0$

2. Vectors

- Two and Three Dimensional Vector notation
- Vector Components
- Length or norm of a vector $|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$
- Addition of Vectors and multiplication by scalars
- Standard Basis Vectors
- Normalization of Vectors (creating unit vectors) $\mathbf{a}/|\mathbf{a}|$
- Force Problems

3. The Dot Product

- Definition
- Angle between vectors $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$
- Orthogonal (Perpendicular) Vectors $\mathbf{a} \cdot \mathbf{b} = 0$
- Direction Angles and Direction Cosines
- Scalar and Vector Projections $\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$, $\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$
- Work $w = \mathbf{F} \cdot \mathbf{D}$

4. The Cross Product

- Definition $\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- Determinants (2x2, 3x3)
- Magnitude and Direction of Cross Product $|\mathbf{a} \times \mathbf{b}| = ab \sin \theta$
- Cross Product of Parallel Vectors
- Area of Parallelogram = $|\mathbf{a} \times \mathbf{b}|$
- BAC-CAB Rule and other properties
- Scalar Triple Product and Volume of Parallelepiped $V = |\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}|$
- Torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

5. Equations of Lines and Planes

- Parametric and Symmetric Equations for a Line $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$
- Vector and Scalar Equations of Planes $\mathbf{N} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 \Rightarrow ax + by + cz = d$
- Determination of Lines and Planes given a variety of information.
- Parallel/Perpendicular Lines or Planes or Angle between planes.
- Distance of a Point to a Plane

6. Cylinders and Quadric Surfaces

- Recognize Various Surfaces – Cylinders and Quadrics
- Slices: Intersection of Surfaces by planes parallel to the coordinate planes
- Parabolic-Circular-Elliptic Cylinders
- Ellipsoid-Elliptic Paraboloid-Hyperbolic Paraboloid-Cone-Hyperboloids of One and Two Sheets