MAT 261 Exam V

Name ________________________

a. Let $\mathbf{F}(x,y,z) = (x^2+y)\mathbf{i} + (y^2+z)\mathbf{j} + (z^2+x)\mathbf{k}$. Compute

i. Ñ x **F**.

ii. Ñ ´ **F**.

b. State the

i. Fundamental Theorem of Calculus.

ii. Fundamental Theorem for line integrals.

c. Find the area of the triangle with vertices at (1,0,0), (0,2,0),(0,0,3) using integration. [Sketch the triangle.]

Instructions:

a. Do all of your work in this booklet. Do not tear off any sheets.
b. **Show all of your steps** in the problems for full credit.
c. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
d. Place a **box around your answers**.
e. Place your **name on all of the pages**.
f. If you need more space, you may use the back of a page and write **On back of page #** in the problem space**.**

DO AS MANY PROBLEMS AS YOU CAN!

Page	**Pts**
1 (35 pts)	
2 (35 pts)	
3 (30 pts)	
Total (100 pts)	

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d. Let $\nabla^2 f = 0$ in a region $D \subset R^2$. Prove using Green's Theorem that $\int_{\partial D} \frac{\partial f}{\partial y}dx - \frac{\partial f}{\partial x}dy = 0$.

e. Let D be the region between $y = x^2$ and $x = y^2$ for $0 \le x \le 1$. Compute $\int_C x^2 y\,dy + xy^2\,dx$ for C the boundary of the region followed counterclockwise.

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a. Consider the surface:
$\mathbf{r}(u,v) =< v\cos u, v\sin u, 2v >$, $0 \le v \le 2$, $0 \le u \le 2\pi$.

i. Describe and/or draw the surface.

ii. Find a normal to the surface at $(u,v) = (0,1)$.

iii. What is the equation of the tangent plane at this point?

iv. What is the area of this surface?

Let $\mathbf{F}(x,y) =< e^x\cos 3y, -3e^x\sin 3y >$.
b. Find a function $f(x,y)$ such that Ñ$f = \mathbf{F}$ for all (x,y).

c. Evaluate $\int \mathbf{F}\cdot d\mathbf{s}$ for the path $\mathbf{c}(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$.

d. Compute Ñ $\times\,\mathbf{F}$.

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a. Sketch the vector field $\mathbf{F} = \dfrac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2 + y^2}}$

b. Compute $\int_C y\, ds$ for C the upper half of the circle $x^2 + y^2 = 9.$.

c. Is $\int_C (2y^2 - 12x^2y^3)\, dx + (4xy - 9x^4y^2)\, dy$ path independent? Why?

d. Use Gauss' Theorem to prove that $\iint_S (f\nabla g - g\nabla f) \cdot \mathbf{n}\; dS = \iiint_E (f\nabla^2 g - g\nabla^2 f)\; dV$

e. Use Stoke's Theorem to evaluate: $\int_C y\, dx + (2x - z)\, dy + (z - x)\, dz,$ where C is the intersection of $x^2 + y^2 + z^2 = 4$ and $y = 1.$

f. Prove $\nabla \times (\nabla f) = 0.$