

MAT 261 Exam IV

Name _____

1. Compute $\iint_R y \sin(\pi x) \, dx dy$ for $R = [0, 1] \times [1, 2]$.

2. Sketch the region of integration, interchange the order, and evaluate: $\int_0^4 \int_{\sqrt{x}}^2 x^2 y \, dy dx$.

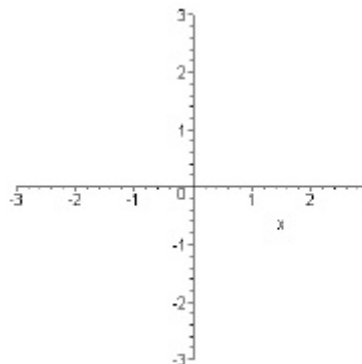
3. Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} x \, dx dy$ using polar coordinates.

Instructions:

- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Place a **box around your answers**.
- Place your **name on all of the pages**.
- If you need more space, you may use the back of a page and write **On back of page #** in the problem space.

Page	Pts
1 (25 pts)	
2 (25 pts)	
Total (50 pts)	

4. Find the image of the unit square under the transformation $T(u, v) = (u - 2v, u + v)$.



What is the Jacobian of this transformation?

5. Find the surface area of the part of the plane $2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$.

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6. A region of space lies between two surfaces, which are given by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

a) Sketch the region and find the equation for the curve at the intersection.

b) Express the volume of the region as a triple integral in Cartesian coordinates. **Do not perform the integration!**

c) Express the volume of the region as a triple integral in cylindrical coordinates and carry out the integration.

7. Evaluate $\iint_D \frac{2y}{x^2+1} dA$ for the region given by

$$D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}\}.$$

8. A hollow wooden bowl of uniform density D has the shape of a hemisphere of outer radius 10" and an inner radius of 9". The center of mass is located at $(0, 0, \bar{z})$. Set up the integral expression for the mass and \bar{z} .

BONUS: Compute \bar{z} .