MAT 261 Exam IV

Name _____

1. Compute $\iint_R y\sin(\pi x)\,dxdy$ for $R = [0,1]\times[1,2]$.

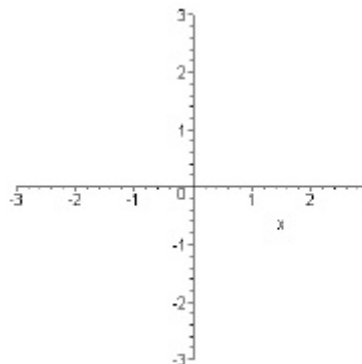2. Sketch the region of integration, interchange the order, and evaluate: $\int_0^4 \int_{\sqrt{x}}^2 x^2 y\,dydx$.

3. Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} x\,dxdy$ using polar coordinates.

Page	Pts
1 (25 pts)	
2 (25 pts)	
Total (50 pts)	

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4. Find the image of the unit square under the transformation $T(u,v) = (u-2v, u+v)$.

What is the Jacobian of this transformation?

5. Find the surface area of the part of the plane $2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$.

6. A region of space lies between two surfaces, which are given by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

a) Sketch the region and find the equation for the curve at the intersection.

b) Express the volume of the region as a triple integral in Cartesian coordinates. **Do not perform the integration!**

c) Express the volume of the region as a triple integral in cylindrical coordinates and carry out the integration.

7. Evaluate $\iint\limits_{D} \frac{2y}{x^2+1} dA$ for the region given by

$D = \{(x, y) | 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$.

8. A hollow wooden bowl of uniform density D has the shape of a hemisphere of outer radius 10" and an inner radius of 9". The center of mass is located at $(0, 0, \bar{z})$. Set up the integral expression for the mass and $\bar{z}$.

BONUS: Compute $\bar{z}$.