MAT 261 Exam II   **Name** _____

Problem	Score
1 (12 pts)	
2 (16 pts)	
3 (10 pts)	
4 (12 pts)	
Total (50 pts)	

1. (12 pts) Let $x(t) = 3\sin(\pi t^2)$, $y = 3\cos(\pi t^2)$.

a. Write the Cartesian equation for this curve.

b. Determine the slope of this curve at $t = \dfrac{1}{2}$.

c. What is the length of the curve for $0 \le t \le 1$?

d. Determine the equation of the tangent line to the curve at the point where $t = \dfrac{1}{2}$.

2. (16 pts) Consider the curve with the position as a function of time given by
$$\mathbf{r}(t) = <3\sin 2t, -1, 3\cos 2t>.$$
a. Find the velocity.

b. Find the acceleration.

c. Find the speed.

d. Find the tangent to the curve, $\mathbf{T}(t)$, at $t = \pi$.

e. Find the normal to the curve, $\mathbf{N}(t)$, at $t = \pi$.

f. Find the binormal vector, $\mathbf{B}(t)$, at $t = \pi$.

g. Determine the curvature at $t = \pi$.

h. Find the arclength function.

Bonus What is the centripetal acceleration, a_N, in Problem 2 at $t = \pi$?

3. (10 pts) Motion Problems

a. A particle starts at the origin with initial velocity $\mathbf{v}(0) = 2\mathbf{k}$. If it's acceleration is $\mathbf{a} =< 1, t, 0 >$, then what is the particle's position at $t = 1.0$?

b. A soccer ball is kicked with initial speed 25.0 m/s at an angle of $20.0°$. [Answer to three decimal places.]

i. What is its time of flight?

ii. What is the range?

iii. What is the maximum height?

4. (12 pts) Hodge Podge

a. Let $\mathbf{L} = \mathbf{r} \times \mathbf{v}$, where $\mathbf{r}$ is the position and $\mathbf{v}$ the velocity. Compute and simplify:
$$\frac{d\mathbf{L}}{dt} =$$

b. Prove $\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \frac{d\mathbf{u}(t)}{dt} \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \frac{d\mathbf{v}(t)}{dt}$

c. Find the equation of the osculating plane at $(0,2,1)$ for the curve $x = \ln t, y = 2t, z = t^2$.

d. Kepler's Third Law is given by $T = \frac{2\pi}{\sqrt{GM}} a^{3/2}$. What are T and a?

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Bonus

If $\sqrt{GM} = 2.0 \times 10^7 \, \mathrm{m}^{2/3} \, \mathrm{s}^{-1}$ for the moon circling the Earth, then how far on average is the moon from the Earth assuming one month is 28 days?