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a. Let $\mathbf{v} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$, and $\mathbf{w} = \mathbf{i} - \mathbf{k}$.

i. Find the area of the parallelogram formed by the vectors **v** and **w**.

ii. Compute $\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})$

iii. Find the angle between **u** and **w**.

iv. Determine the parametric equations for the line through (1,2,3) in the direction of **v**.

b. Let a particle have an acceleration of $\mathbf{a} = t\mathbf{i} - 5\mathbf{j}$ and at the origin have an initial velocity of $\mathbf{v}_0 = \mathbf{i} + \mathbf{j}$. Determine the particle's position and velocity at time $t = 2$?

c. State the Fundamental Theorem of Line Integrals.

d. Suppose $f(x, y) = x^2 y + \ln y$ and $x = s^2 + t^2,\ y = st$. Find $\dfrac{\partial f}{\partial s}$ at $(s, t) = (1, 2)$.

e. Let $f(x, y) = y^2 \sin(2x)$. Compute df in terms of dx and dy.

f. Define $\mathbf{F}(x, y, z) = (x^2 + y)\mathbf{i} + (y^2 + z)\mathbf{j} + (z^2 + x)\mathbf{k}$.

i. Compute $\nabla \cdot \mathbf{F}$

ii. Compute $\nabla \times \mathbf{F}$.

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a. Consider a lamina oriented in the *xy*-plane occupying the region that lies between two quarter circles with radii one and two in the first quadrant. If the lamina density is $\rho = x^2 + y^2$, what is its mass?

b. Evaluate $\iint_D 3xe^{x-3y}\, dxdy$ over the region bounded by the lines $2x+3y=1,\ 2x+3y=0,$ $x-3y=1,\ x-3y=0$ using the transformation $u=2x+3y,\ v=x-3y.$

c. Sketch and label several level curves for $f(x,y)=6xy$.

d. Consider the cone $\mathbf{r}(u,v) = < u\cos v, u\sin v, u >$ for $0 \le u \le 1,\ 0 \le v \le 2\pi$. Find its surface area.

e. Fido is on a mountain which has a shape described by $z = 100 - 25x^2 - 4y^2$. If Fido is at the point (1, 2, 59) on the mountain, in what the direction must Fido go to get down the mountain as fast as possible?

f. Evaluate $\int_0^1 \int_y^1 \sin x^2\, dxdy$ by changing the order of integration.

g. Let $f(x,y) = x^2 + 2xy^2$. What is the rate of change of $f(x,y)$ in the direction $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$ at $(x,y) = (-1,1)$?.

h. Find the length of the curve $\mathbf{r}(t) = < t^2, 2t, \ln t >$ for $1 \le t \le e$.

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a. Let D be the region between $y = x^2$ and $x = y^2$ for $0 \le x \le 1$ and C the curve enclosing D. Compute $\int_C x^2 y\,dy + xy^2\,dx$.

b. Find $\frac{\partial z}{\partial x}$ when $xz = \ln(yz)$.

c. Find and classify the relative extremum for $f(x,y) = 2x - x^2 + y^2 - 1$.

d. Find the extreme values of $f(x,y) = x + y$ on the curve $2x^2 + y^2 = 1$.

e. Let $\mathbf{F}(x,y) = (e^x \cos 3y)\mathbf{i} - (2y + 3e^x \sin 3y)\mathbf{j}$.

i. Find a function $f(x,y)$ such that $\nabla f = \mathbf{F}$.

ii. What is $\nabla \times \mathbf{F}$?

f. Find the work done, $W = \int_C \mathbf{F} \cdot d\mathbf{r}$, by the force field $\mathbf{F}(x,y) = x^2\mathbf{i} - xy\mathbf{j}$ in moving a particle along the quarter circle $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$, $-\frac{\pi}{2} \le t \le 0$.

g. For a general parametrized curve $\mathbf{r}(t)$, define the following using $\mathbf{r}$ and its derivatives:

i. Tangent vector.

ii. Normal vector.

iii. Curvature.

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a. A hole of radius 1/2 is drilled vertically through the center of the hemisphere $x^2+y^2+z^2=1,\ z\ge 0.$ Determine the volume integral of the remaining piece using cylindrical coordinates.

b. Consider the surface $z=2x^2-3y^2.$

i. Find a normal vector at the point (-2,1).

ii. What is the equation of the tangent plane (-2,1)?

c. Use Gauss' Divergence Theorem to evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S},$ where S is the positively oriented surface of the unit sphere with its center at the origin and $\mathbf{F}(x,y,z)=(z-y)\mathbf{i}+(y^2+z^2)\mathbf{j}+(y-x^2)\mathbf{k}.$

d. Use Stoke's Theorem to evaluate: $\int_C \mathbf{F}\cdot d\mathbf{r}=\int_C y\,dx+(2x-z)\,dy+(z-x)\,dz,$ for C the intersection of $y=\sqrt{x^2+z^2}$ and $y=1.$

e. The volume of a right circular cone of height H and radius R is $V=\frac{1}{3}\pi R^2H.$ If the radius and height are measured as $R=3.2\pm 0.2$ in and $H=5.6\pm 0.1$ in, what is the error is made in computing the volume?

f. Find the equation of the plane containing the points (1,1,0), (2,0,-1) and (0, 3, 0).

g. Name the surface:

i. $3x^2+y=5.$ ________________

ii. $z=3x^2-y^2.$ ________________

iii. $x^2+y^2-z^2=-5.$ ________________