Review for Exam III

1. Parametric Equations
   a. Determining graphs of $x = f(t),\ y = g(t),\ a \le t \le b$
   b. Special Curves:
      i. Circle - $x = r\cos t,\ y = r\sin t,\ 0 \le t \le 2p$ and variations
      ii. Ellipse - $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2p$
      iii. Hyperbola - $x = a\cosh t,\ y = b\sinh t,\ -\infty < t < \infty$
   c. Slope of Tangent Line - $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$.
   d. Arclength

$$ds^2 = dx^2 + dy^2 = \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)dt^2 \Rightarrow L = \int ds = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt\ .$$

   e. Area – Surface of Revolution

$$A = \int_{s(a)}^{s(b)} x\,ds = \int_a^b f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt\ ,$$

$$A = \int_{s(a)}^{s(b)} y\,ds = \int_a^b g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt\ .$$

2. Polar Coordinates
   a. Conversion Between Cartesian and Polar Coordinates
   b. Graphing Polar Equations:
      $r = a, q = a, r = a\cos(q), r = a\sin(q), r = \cos(nq), r = \sin(nq)$, etc.
   c. Slope of Tangent Line - $\dfrac{dy}{dx} = \dfrac{f'(q)\sin q + f(q)\cos q}{f'(q)\cos q - f(q)\sin q}$, $r = f(q)$.
   d. Area Between Polar Curves - $A = \dfrac{1}{2}\int_a^b [f(q)]^2\,dq$,

$$A = \frac{1}{2}\int_a^b [f(q)]^2\,dq - \frac{1}{2}\int_a^b [g(q)]^2\,dq$$

   e. Arclength $ds = \sqrt{\left(\dfrac{dx}{dq}\right)^2 + \left(\dfrac{dy}{dq}\right)^2}\,dq = \sqrt{r^2 + \left(\dfrac{dr}{dq}\right)^2}\,dq$, so

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{dq}\right)^2}\ dq$$

3. Conics
   a. Parabola - $y^2 = 4px$, focus $(p,0)$, directrix $x = -p$
   b. Ellipse - $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, foci $(\pm c,0)$, vertices $(\pm a,0)$, $c^2 = a^2 - b^2$, etc.

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c. Hyperbola - $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, foci $(\pm c,0)$, vertices $(\pm a,0)$, $c^2=a^2-b^2$, asymptotes, $y=\pm\frac{b}{a}x$ etc.

d. Polar Form - $r=\frac{ed}{1\pm e\cos q}$ or $r=\frac{ed}{1\pm e\sin q}$, where
circle - $e=0$, ellipse - $e<1$, parabola - $e=1$, hyperbola - $e>1$