Review for Exam II

I. First Order Differential Equations

a. Separation of Variables

i. $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx + C$

ii. General Solutions – Implicit and Explicit

iii. Initial Value Problems – Particular Solutions

b. Linear Differential Equations

i. Find integrating factors and solve initial value problems

ii. $y' + P(x)y = Q(x)$

$$I(x) = \exp \int^{x} P(x)\,dx \Rightarrow (Iy)' = IQ$$

$$y(x) = \frac{1}{m(x)}\left[\int^{x} m(t)f(t)\,dt + C\right]$$

c. Autonomous Equations $\frac{dy}{dx} = f(y)$

i. Equilibrium (constant) solutions $f(y_0) = 0.$

d. Direction Fields

e. Euler's Method $y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}),\ \ y_0 = y(x_0)$ given

II. Second Order Differential Equations

a. Homogeneous, Constant Coefficient Equations $ay'' + by' + cy = 0$

b. Solutions - $y(x) = e^{rx},\ ar^2 + br + c = 0.$

i. Two, real distinct solutions $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

ii. One real solution $y = (c_1 + c_2 x)e^{rx}$

iii. Two complex conjugate solutions $y = (c_1 \cos b\,x + c_2 \sin b\,x)e^{a\,x}$

III. Applications

i. Free Fall $y'' = -g$

ii. Orthogonal Trajectories $\left.\frac{dy}{dx}\right|_{\text{New}} = -\frac{1}{\left.\frac{dy}{dx}\right|_{\text{Old}}}$

iii. Growth and Decay $y(t) = Ae^{kt}$

1. Populations
2. Radioactivity – Half Life

iv. Newton's Law of Cooling $\frac{dT}{dt} = k(T - T_s),\ T(t) = T_s + (T_0 - T_s)e^{kt}$

v. Mixing Problems

1. Rate of Change of Quantity = Rate In – Rate Out

vi. Population Models – Logistic Model

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right),\ \ P(t) = \frac{K}{1 + Ae^{-kt}},\ A = \frac{K - P_0}{P_0}$$

vii. Mass-Spring Oscillation $x'' + w_0{}^2 x = 0.\ w_0 = \sqrt{\frac{k}{m}}.$