Review for Exam I

Assorted Derivatives

$\frac{dc}{dx}=0$	$\frac{d}{dx}(x^n)=nx^{n-1}$	$\frac{d}{dx}(e^{ax})=ae^{ax}$	$\frac{d}{dx}\left(a^x\right)=(\ln a)a^x$	$\frac{d}{dx}(\ln x)=\frac{1}{x}$
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\frac{d}{dx}(\sec x)=\sec x\tan x$
$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(\sinh x)=\cosh x$	$\frac{d}{dx}(\cosh x)=\sinh x$	$\frac{d}{dx}(\tanh x)=\text{sech}^2 x$	$\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}$
$(u+v)'=u'+v'$	$(uv)'=uv'+u'v$	$\left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$	$(f\circ g)'=f'[g]\cdot g'$	$[f(g(x))]'=f'(g(x))g'(x)$

Note: The Derivative $\frac{d}{dx}\sqrt{x}=\frac{1}{2\sqrt{x}}$ occurs often.

Integrals

I. **Fundamental Theorem of Calculus:** For any continuous function f on $[a,b]$ with

$$F'(x)=f(x),\quad \int_a^b f(x)=F(b)-F(a).$$

II. Integrals You Should Know

$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq -1$	$\int \frac{1}{x}dx=\ln\|x\|+C$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+C$	$\int a^x\,dx=\frac{a^x}{\ln a}+C$
$\int \sin ax\,dx=-\frac{1}{a}\cos ax+C$	$\int \cos ax\,dx=\frac{1}{a}\sin ax+C$	$\int \sec^2 ax\,dx=\frac{1}{a}\tan ax+C$	$\int \csc^2 ax\,dx=-\frac{1}{a}\cot ax+C$
$\int \sec x\tan x\,dx=\sec x+C$	$\int \csc x\cot x\,dx=-\csc x+C$	$\int \sinh ax\,dx=\frac{1}{a}\cosh ax+C$	$\int \cosh ax\,dx=\frac{1}{a}\sinh ax+C$
$\int \frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+C$	$\int \tan x\,dx=\ln\|\sec x\|+C$	$\int \cot x\,dx=\ln\|\sin x\|+C$
$\int \ln x\,dx=x\ln x-x+C$	$\int \sec x\,dx=\ln\|\sec x+\tan x\|+C$	$\int \csc x\,dx=-\ln\|\csc x-\cot x\|+C$	$\int \frac{dx}{ax-b}=\frac{1}{a}\ln\|ax-b\|+C$

III. Methods of Integration

a. Integration by Parts

$$\int u\,dv=uv-\int v\,du$$

b. Trigonometric Integrals – Odd Powers of sine and cosine

$$\int \cos^3 x\,dx=\int \cos^2 x\cos x\,dx=\int (1-\sin^2 x)\cos x\,dx=\int (1-u^2)\,du \text{ for } u=\sin x$$

Need $\sin^2 x+\cos^2 x=1$

c. Trigonometric Integrals – Even Powers of sine and cosine

$$\int \cos^2 x\,dx=\int \frac{1}{2}(1+\cos 2x)\,dx,\ \int \sin^2 x\,dx=\int \frac{1}{2}(1-\cos 2x)\,dx$$

d. Trigonometric Substitution

$\sqrt{a^2-x^2}$	$x=a\sin q$	$dx=\cos q\,dq$
$\sqrt{a^2+x^2}$	$x=a\tan q$	$dx=\sec^2 q\,dq$
$\sqrt{x^2-a^2}$	$x=\sec q$	$dx=\sec q\tan q\,dq$

Review for Exam I

e. Rational Functions

$$\int \frac{dx}{(x-a)(x-b)} = \int \left[\frac{A}{x-a} + \frac{B}{x-b} \right] dx$$

$$\int \frac{dx}{(x-a)^2(x-b)} = \int \left[\frac{A}{(x-a)^2} + \frac{B}{x-a} + \frac{C}{x-b} \right] dx$$

$$\int \frac{dx}{(x^2+a)(x-b)} = \int \left[\frac{Ax+B}{x^2+a} + \frac{C}{x-b} \right] dx$$

$$\int \frac{dx}{(x^2+a)^2(x-b)} = \int \left[\frac{Ax+B}{(x^2+a)^2} + \frac{Cx+D}{x^2+a} + \frac{E}{x-b} \right] dx$$

Partial Fractions:

$\frac{A}{x-a} + \frac{B}{x-b} = \frac{A(x-b)+B(x-a)}{(x-a)(x-b)}$ and compare numerators to get constants.

f. Rationalizing Substitutions

Replace radical: $u = \sqrt{x+a}$, $du = \frac{dx}{2\sqrt{x+a}} = \frac{dx}{2u}$, $x = u^2 - a$

g. Improper Integrals

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx, \quad \int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx,$$

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^0 f(x)\,dx + \int_0^\infty f(x)\,dx$$

P-Test: $\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$.

If $f(x)$ has a discontinuity at c, $a \le c \le b$, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{t\to c^-}\int_a^t f(x)\,dx + \lim_{t\to c^+}\int_c^b f(x)\,dx$$

h. Numerical Integration – only for labs and homework

IV. Applications of Integration

a. Arc Length – for curve *y = f(x)* from *x = a* to *x = b*:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

b. Areas of Surfaces of Revolution

Rotation about *x*-axis: $S = \int 2p\, y\, ds$

Rotation about *y*-axis: $S = \int 2p\, x\, ds$,

where $ds = \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\,dx$ for $y = y(x)$, or, $ds = \sqrt{1 + \left[\frac{dx}{dy}\right]^2}\,dy$ for $x = x(y)$

c. Hydrostatic Pressure and Force: $P = r g d$, $F = PA$

d. Center of Mass: $\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$, $M_y = \sum_{i=1}^n m_i x_i$, $M_x = \sum_{i=1}^n m_i y_i$

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx, \quad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx,$$