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Instructions:

- Do all of your work in this booklet.
- Show all of your steps in problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Place your answers in a box.
- If you need more space, you may use the back of the page and write **On back Page #** in the problem space.

1. (3 pts) Find surface area generated by rotating the line $y = 1 - x$ about the x axis for $0 \leq x \leq 1$.

2. (5 pts) For each equation indicate the type of graph you would expect from the list. Some answers may appear twice in your list.

- (A) Line (B) Parabola (C) Rose
 (D) Circle (E) Ellipse (F) Hyperbola

a. $2x^2 - 3y^2 = 12$ _____

b. $r = 3 \cos 2\theta$ _____

c. $x = 2 \sin \theta, y = 2 \cos \theta$ _____

d. $x = 2t^2 + 3, y = 1 - t$ _____

e. $r = \sin \theta$ _____

3. (3 pts) Find the arc length of the curve $f(x) = \ln(\cos x)$ for

$0 \leq x \leq \frac{\pi}{3}$.

4. (18 pts) Evaluate the following:

a. $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$.

b. $\int x^2 e^{2x} \, dx$

c. $\int \tan^3 \theta \, d\theta$.

d. $\int_0^1 \frac{x}{\sqrt{1+3x^2}} \, dx$.

e. $\int \frac{1}{x^2 + 3x - 4} \, dx$.

f. $\int_{-3}^3 \sqrt{9-x^2} \, dx$

5. (2 pts) Determine if the integral $\int_1^{\infty} \frac{x}{1+x^2} dx$ converges or diverges.

6. (3 pts) You initially have 200 g of bacteria. If the population triples every 20 minutes, what is the population after 50 minutes?

7. (4 pts) Consider the curve $x = 1 - t^2$, $y = t^3$ for $0 \leq t \leq 2$.

a. Find the slope of the tangent line at the point with $t = 1$.

b. What is the length of this curve for $0 \leq t \leq 2$?

8. (3 pts) Consider the initial value problem

$\frac{dy}{dx} = x - 4y$, $y(0) = 1$. Use Euler's Method to find an approximation to $y(1)$ using steps of $h = 0.25$.

9. (12 pts) Find the general solution to each equation. If initial conditions are given, then find the particular solution.

a. $\frac{dy}{dx} = xy$.

b. $y' + 2y = e^x$, $y(0) = 3$.

c. $y'' + 2y' + 2y = 0$.

d. $y'' - 2y' - 8y = 0$, $y(0) = 1$, $y'(0) = 0$.

10. (3 pts) A 50 thousand gallon lake is full of polluted water. Pure water is pumped into the lake at a rate of 2000 gal/hr. The well-mixed contents of the lake are pumped out into a filtering system at 2000 gal/hr. How long will it take to reduce the amount of pollutant to one tenth the initial amount?

11. (3 pts) Put the conic $x^2 + 2y^2 + 6x - 8y + 13 = 0$ into standard form. Describe the conic as completely as you can.

12. (3 pts) Consider the curve $r = \sin 3\theta$. Find the area of one of the loops in the curve

13. (9 pts) Sum the series:

a. $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n} =$

b. $\sum_{n=1}^{\infty} \left[\frac{1}{3^n} - \frac{1}{4^n} \right] =$

c. $\sum_{n=1}^{\infty} \frac{2}{n(n+1)} =$

14. (6 pts) Find the limit $\lim_{n \rightarrow \infty} a_n$ to determine if the given sequence converges, or diverges.

a. $a_n = \frac{n^3 - 2n}{n^2 + 5n}$.

b. $a_n = \sqrt[n]{2(n+1)^2}$.

c. $a_n = \left(\frac{2n}{2n-1} \right)^n$.

15. (8 pts) Determine if the following series converge absolutely, converge conditionally, or diverge. State the test used.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$.

b. $\sum_{n=1}^{\infty} \frac{1+n^2}{n^2+n^3}$.

c. $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$.

d. $\sum_{n=1}^{\infty} n^2 e^{-n}$.

16. (3 pts) Use an appropriate series expansion to approximate the following to three decimal places:

$$\int_0^{0.25} \frac{dx}{1+x^2} \approx$$

17. (6 pts) Determine the interval of convergence and radius of convergence of the following power series.

a. $\sum_{n=2}^{\infty} \frac{(x-1)^n}{2^n}$.

b. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2 n!}$.

18. (6 pts) Provide the terms up to and including powers of x^4 in a Maclaurin series expansion of the given function.

a. $f(x) = \cos 2x$.

b. $f(x) = \ln(1-x^2)$.