Column	Points	Score
1	14	
2	14	
3	13	
4	9	
Total	50	

Instructions:
1. Do all of your work in this booklet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may **use the back of the page** and write **On back Page #** in the problem space.

1. Definitions (3 pts)

a. When is the equation $a(x)y'' + b(x)y' + c(x)y = f(x)$ *homogeneous*?

b. Write the *natural frequency* of oscillation in terms of the mass and spring constant.

c. Define the *carrying capacity* of a population?

2. Classifying Equations (6 pts) Classify the following equations by checking all that apply. Answer Y/N if the given equation is linear, first order, or separable. Leave no blanks

Equation	1st Order?	Linear?	Separable?
$y' = xy + x$			
$yy' = x + y$			
$xy'' + y = 2$			
$y' = y^2 \ln x$			

3. (5 pts) You initially have 90 g of a radioactive substance with a half life of 3 months. How much is left after 10 months?

4. ODEs (14 pts) Find the general solution to each equation. If initial conditions are given, then find the particular solution.

a. $\dfrac{dy}{dx} = y \sin 2x.$

b. $\dfrac{dz}{dt} + tz = 0, \ z(0) = 3.$

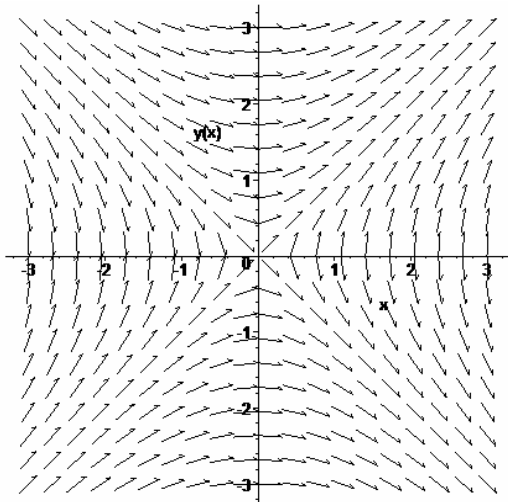c. $y' + 2y = e^{-x}, \ y(0) = 5.$

d. $xy' + y = 2x$

5. (6 Pts) Second Order Equations: Write down the general solution for the following:

a. $y'' + 2y + 2 = 0$.

b. $y'' - 2y - 8 = 0$.

6. (4 pts) Euler's Method: Consider the initial value problem $y' = -2y + x$, $y(0) = 0$. Use Euler's Method to find an approximation to $y(2)$ using steps of $h = 0.5$.

7. (3 pts) Direction Fields: For the given direction field sketch solutions passing through $y(0) = 2$, $y(0) = -1$, $y(-2) = 0$.

8. Equilibrium Solutions (3 pts) Find the *equilibrium solutions* for the differential equation: $\dfrac{dy}{dx} = 3(y^2 - 1)(y + 2)$.

9. Mixing Problem (6 pts) A 40 gallon tank is full of pure water. Brine with a salt concentration of 5.0 lb/gal is pumped into the tank at a rate of 2 gal/hr. The well-mixed contents of the tank are pumped out at 2 gal/hr.

i) Write the differential equation for the system.

ii) Solve the equation using the initial condition.

iii) How much salt is in the tank after 5 hours of pumping?

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Bonus Give the solution to Newton's law of cooling in terms of the initial temperature, the air temperature and the cooling constant.