# Lab 3

# Surfaces of Revolution

## Purpose

To visualize and compute the area of some surfaces of revolution.

## Files

**Surfaces.MW** - *Optional*

## 3.1 Introduction

Until now you have been taught that the definite integral, $\int_a^b f(x)\,dx$, represents the area under the curve $y = f(x)$. However, there are many other applications of integration. In this lab you will explore surfaces of revolution and use integration to determine surface areas.
A surface of revolution is formed when one rotates a given curve about a line, called an axis of rotation. In the Figure on the right, the function $y = \sqrt{x}$ is rotated about the $x$-axis for $x \in [0, 2]$. The original function is seen embedded in the resulting surface of revolution. In this lab you will explore other surfaces of revolution that result from a rotation about the $x$-axis.

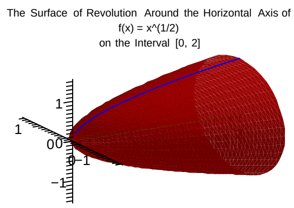


**Figure 3.1:** A surface of revolution generated by rotating $y = \sqrt(x)$ about the $x$-axis for $x \in [0, 2]$.

In the next figure, the function $y = \sqrt{x}$ is rotated about the $y$-axis for $y \in [0, 2]$. Again, you can see the original function embedded in the surface of revolution.
Again, you can see the original function embedded in the surface of revolution. Note the differences between the two surfaces. You could try to imagine how the surface might change if you made the curve longer or shorter. Think about what shapes would result if you changed the function. In fact, one of the main goals of this lab is to learn to visualize surfaces of revolution.

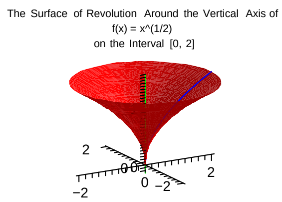


**Figure 3.2:** A surface of revolution generated by rotating $y = \sqrt(x)$ about the $y$-axis for $y \in [0, 2]$.

The other goal in this lab is to use the computer to compute the surface area of a surface of revolution. You will learn in that in principle the surface area can be computed using integration.

We will not derive the surface area formulae at this time. That will be covered in the lecture. However, we can provide a general picture of what is done with the derivation. When looking at the Surface of Revolution Maplets, you should explore the feature that allows you to view a plot of the frustra.

The way that one computes surface areas is to approximate the surface of revolution by many thin strips, or bands. Each band area is approximated by the surface area of a frustrum, or piece of a cone. For a revolution about the $x$-axis, we show one set of frustra to the right. Summing over all such bands and taking the limit as the bands become thinner, one can show that the surface area for a surface formed by revolving $y = f(x)$ about the $x$-axis for $a \leq x \leq b$ is given by

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx. \tag{3.1}$$

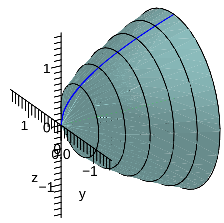

**Figure 3.3:** Approximating the surface of revolution by frustra.

Similarly, if a surface is formed by revolving $x = g(y)$ about the $y$-axis for $c \leq y \leq d$, the surface area is

$$S = \int_c^d 2\pi y\sqrt{1 + [g'(y)]^2}\, dy. \tag{3.2}$$

## 3.2 Visualization

Use Maple to plot the functions below on the given intervals. Mentally visualize the shape of the surface obtained by rotating the curve around the given axis. Describe the shape of the surface and give your best guess for the area of the surface.

1. $y = x$ , for $x \in [0, 1]$; a) About the $x$-axis, b) About the $y$-axis, c) About $x = 2$.
2. $y = x^2$, for $x \in [0, 2]$; a) About the $x$-axis, b) About the $y$-axis.
3. $y = \frac{1}{x}$, for $x \in [1, 20]$, about the $x$-axis.
4. $y = \exp(-x^2)$, for $x \in [0, 2]$; a) About the $x$-axis, b) About the $y$-axis.
5. $y = \sqrt{x}[(x-1)^2 + c]$, for $x \in [0, 1.7]$ and for $c = 1/5, 1/2, 1$, about the $x$-axis.
6. $y = \frac{1}{x^6+1}$, for $x \in [0, 2]$ about the $y$-axis.
7. $y = \sqrt{1 - x^2}$, for $x \in [0, 1]$; a) About the $x$-axis.

You can use Maple to investigate these surfaces using the Surface of Revolution Maplet found under the Tutors section. A window will appear in which you can type in the given functions, change the intervals, display surfaces rotated about horizontal and vertical axes, and even change the number of frustra used. Maple will show the surface integral and Riemann sums.

## 3.3 Integration

For each of the functions listed above, graph and compute the area using the Surface of Revolution Maplet.

Your observations should be written using sentences with completed thoughts. They should address not only the final value for the area, but your observations about the shapes generated and what you saw and learned as you played with the various parameters. You should also think about the relationship between the curve and the surface as you visualize the surfaces. Imagine the curve rotating to form the surface.

Note: if you prefer to use straight Maple 10 worksheets, download the file **Surfaces.MW**