Lab 8

Roses are Red ...

Purpose

To become familiar with the relationship between polar and Cartesian coordinates and to learn to graph functions of the type $r = f(\theta)$.

8.1 The Polar Coordinate System

In this lab you will experiment with a two-dimensional coordinate system different from the Cartesian (x, y) system. In two-dimensional polar coordinates, a point P is determined by the coordinates (r, θ). r is the distance from the point P to the origin, and θ is the angle which the ray OP makes with the positive $x-$axis. By convention, an angle which sweeps in a counter-clockwise direction from the positive $x-$axis is given a positive sign, and an angle which sweeps in a clockwise direction is given a negative sign. **Note:** θ is typically given in *radians*.
We can easily relate polar and Cartesian coordinate systems using a bit of trigonometry. From the figure on the right, we see there are natural relationships between the x and y coordinates and the polar coordinates using what we know about right triangles. The polar and Cartesian coordinate systems are related by the following equations:

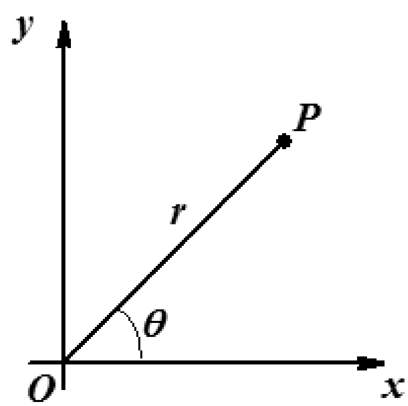

Figure 8.1: Polar Coordinates

$$x^2 + y^2 = r^2 \qquad \tan\theta = \frac{y}{x} \tag{8.1}$$

$$x = r\cos\theta \qquad y = r\sin\theta. \tag{8.2}$$

Consider, for example, the point P with Cartesian coordinates $(5, -5)$. In your lab notebook,

sketch the graph and derive the following results: The distance r from P to the origin is $5\sqrt{2}$. The angle that the ray OP makes with the positive $x-$axis is $-\pi/4$ radians ($-45°$). Thus for P, $(r, \theta) = (5\sqrt{2}, -\pi/4)$.
If you wanted to plot the function $r = r(\theta)$, you could convert to a parametric set of equations for $x(\theta)$, and $y(\theta)$) and plot the corresponding points on a standard Cartesian grid. However, it would be more natural to plot the function $r(\theta)$ by getting $r-$ values for various $\theta-$values and then plotting the point (r, θ). This is not easy to do on a Cartesian coordinate system; i.e., the standard $x-$ and $y-$ axes. As you know, this system consists of a grid of intersecting lines x =constant and y =constant. The system needed for plotting polar equations consists of intersecting curves r =constant and θ =constant as depicted in the figure on the right. We will use this *Polar coordinate system* in the lab.

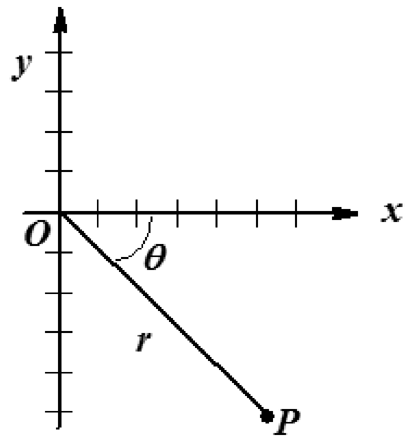

Figure 8.2: Polar Coordinates

Instructions

We can easily plot polar equations in Maple. The simplest polar equation is of the form $r = r(\theta)$. We can plot this parametrically like we had in the last lab by using the standard Cartesian axes using

$$x(\theta) = r(\theta)\cos(\theta) \qquad \text{and} \qquad y(\theta) = r(\theta)\sin(\theta). \tag{8.3}$$

Quicker methods would be to type **plot([1,theta,theta=0..2*Pi],coords=polar);** or **polarplot(1)** Both of these commands plot $r(\theta) = 1$ for $0 \leq \theta \leq 2\pi$.

You can use the last method in the following to plot $r = f(\theta)$ for $a \leq \theta \leq b$. However, you should first enter

restart: with(plots):

Then you can enter **polarplot(f(theta),theta=a..b);**

- For $0 \leq \theta \leq 2\pi$ plot the polar equation

$$r(\theta) = 4\cos(2\theta). \tag{8.4}$$

  What do you see?

- Now, you can experiment with the function in (8.4). What happens if the coefficient 4 is changed to 2, 3, and -2? Record your observations.

- Using a coefficient of 2 in (8.4), switch from the cosine to the sine function. What is the effect? Can you explain this?

- Now, switching back to the cosine function, experiment with the coefficient of θ. Try values of 3, 4, and 5.

- Graph the new function

$$r(\theta) = 4 + 2\sin\theta. \tag{8.5}$$

  A graph of the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$ is called a *limaçon*. Experiment with different values of the constant and the coefficient in front of the sine function. Try both positive and negative values. Keep the one that looks like a heart (formally called a *cardiod*) and one that looks like a loop within a loop.

- Now set θ to go from 0 to 6π. Graph $r(\theta) = \theta$. What kind of graph do you get? With the range of θ set back to $[0, 2\pi]$, graph $r(\theta) := 10$. What do you get?

Exercises

▷ **Exercise 8.1** Describe the effects of varying the coefficients for the rose petals. How many petals will the rose have if the coefficient n of θ is an even number? What if n is odd?

▷ **Exercise 8.2** What would be the Cartesian form of the equation $r(\theta) = 10$?

▷ **Exercise 8.3** Graph $r(\theta) = 8 \sin \theta$. What is the center (x, y) and radius of this circle? What would be its equation in Cartesian form?

▷ **Exercise 8.4** Consider the circle given in Cartesian form as $x^2 + (y - 3)^2 = 9$. What are the (x, y) coordinates of its center? What is its radius? Can you guess the polar form of this circle? Check to see if you are right by doing the following: graph your "guess" function and the (x, y) coordinates of the circle and see if the graph is a circle with the correct center and radius.