Lab 6

Damped Oscillations

Purpose

To explore the dynamics of damped harmonic motion using a mass-spring oscillator

6.1 The Mechanical Spring

A common example of a harmonic oscillator is an object of mass m hanging on a spring. As the spring is stretched away from equilibrium, it pulls on the mass, and as the spring is compressed, it pushes. We remark that the mass displaces the natural equilibrium position of the spring. It can be shown that this does not in general affect the analysis below. We simply consider the rest position of the mass as the new equilibrium position. We will also be using special conical shaped springs that have been carefully designed to behave as massless springs; thus, the mass of the springs shall be neglected.

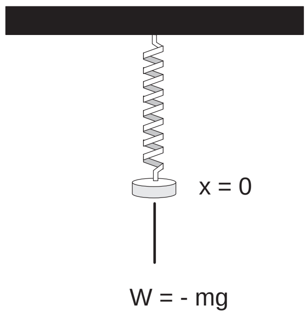

Figure 6.1: Oscillator

The ratio of the restoring force to the displacement, is a constant, called the *stiffness* or the *spring constant*. If the displacement increases by a factor of 2, the force increases in the opposite direction by a factor of 2.
More formally, if F_s is the force exerted by the spring and x the displacement from equilibrium, then

$$F_s = -kx, \tag{6.1}$$

where k is the spring constant. Equation (6.1)is known as Hooke's law. We should also take into account the effect of friction. The simplest model assumes that the frictional force F_f is proportional to the negative of the velocity

$$F_f = -b\frac{dx}{dt}. \tag{6.2}$$

The total Force is $F_s + F_f$. If we apply Newton's second law, we get the equation of motion

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}. \tag{6.3}$$

Dividing the last equation by m and rearranging terms, we get

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0. \tag{6.4}$$

Here, we have written $\beta \equiv b/2m$. Also, we have introduced the quantity

$$\omega_0^2 = \frac{k}{m}. \tag{6.5}$$

In the absence of friction, equation (6.4) reduces to

$$\frac{d^2x}{dt^2} = -\omega_0^2 x. \tag{6.6}$$

which is the equation of a simple harmonic oscillator with natural frequency ω_0. We shall refer to (6.4) as the damped SHO differential equation.

It can be shown that if $\omega_0^2 > \beta^2$, the general solution of the differential equation is of the form

$$x(t) = Ae^{-\beta t}\cos(\omega t - \delta), \tag{6.7}$$

where

$$\omega^2 = \omega_0^2 - \beta^2, \tag{6.8}$$

and A and δ are arbitrary constants.

The quantity ω is called the "frequency" of the oscillator, although, technically, this is an abuse of language, since the motion is not periodic. Also, the "amplitude" $Ae^{-\beta t}$ of the oscillation is a decreasing exponential function, where A is the initial amplitude.

▷ **Exercise 6.1** Verify that (6.7) is a solution of (6.4).

▷ **Exercise 6.2** Plot the given pairs of functions on the same axes. Write a description of these functions, including the values of the frequencies, amplitude behavior, shifts, and zeroes of the oscillatory functions. Describe how the pairs of functions are related.

a) $x = 10e^{-t/4}\cos(10t)$ and $x = 10e^{-t/4}$.

b) $x = 10e^{-t/2}\cos(2t - \frac{\pi}{2})$ and $x = 10e^{-t/2}$.

c) $x = 10e^{-t/4}\cos(10t)\cos(t)$ and $x = 10e^{-t/4}$.

6.2 Mass-Spring Experiment

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Materials

- Distance probe and interface.
- 1 meter rod with clamp.
- 1 metal bar clamp
- 1 set of metric masses
- 1 50g mass holder
- 1 500 g mass plus 250 g holder (optional)

To set up this experiment attach a rod clamp on the edge of a laboratory table. Clamp one of the metal bar hangers to the vertical rod. Hook one of the springs to the metal bar and attach a weight holder to the bottom end.

In the first part of this experiment you will collect data to find the spring constant. To do this you simply measure the displacement of the point of equilibrium of the spring as you vary the mass load. Collect 8 to 10 data points for masses between 50g and 1kg. Do not forget that the mass holder itself has a mass of 50g.

In the second part, you will use the data acquisition system to record the motion of the oscillator. Hang a 0.5 kg mass on the bottom of the spring. Lower the weight gently until the spring is stretched to its new equilibrium position. Use some masking tape to secure the mass safely to the weight holder and to the spring. You do not want the mass slipping off and crushing the distance probe that is directly below the mass. Care must taken to focus the probe onto the the target. Connect the equipment and run a couple of trial measurements of the position of the mass versus time to adjust the ranges of the axes. Record a dozen cycles of the motion and save the data into a spreadsheet.

Instructions

- Using MS Excel, make a plot of deflection versus mass using the data collected in the first part of the experiment. Use linear regression and Hooke's Law to find the value of the spring constant. (Do not forget that the force on the spring, exerted by the mass, is the Weight$=-mg$)

- Plot the position vs time data for the spring oscillation. From your plot, determine the average period of the oscillation. (Recall that the period is the time it takes the mass to complete one cycle.). Use the period to determine the "frequency" $\omega = \frac{2\pi}{T}$.

- Compute the frequency of the oscillation.