Lab 7

Dangerous Curves

Purpose

To explore a variety of parametric curves in the plane.

7.1 Parametric Curves

Most of the graphing, which you have encountered to this point has been that of functions $y = f(x)$. It is natural to think of graphs as sets of points (x, y), where we hope that y is a function of x. In the description of the motion of an object in a plane, we are often interested in the position, the speed, and the acceleration of this object. Such motion is called *curvilinear motion.*

In decribing this curvilinear motion, it is common to express the position (x, y) separately as functions of time. Such equations are given in the form

$$x = x(t), \qquad y = y(t). \tag{7.1}$$

Equations in this form, where two variables are given in terms of a third variable, are known as *parametric equations.* In the above equations, t is referred to as a parameter.

In this lab we will see how some familiar, and some not so familiar, curves can be described parametrically.

Instructions

Before you actually begin to plot the parametric curves below, you should set up a generic worksheet. To do this, follow the steps below:

- Type **restart:**
- Enter the plot command for the parametric functions involved: $x = x(t)$ and $y = y(t)$. In general you will type **plot([x(t),y(t),t=a..b]);** where
  - **x(t)**,**y(t)** are the functions, $x(t)$ and $y(t)$, listed below.
  - **a** and **b** are the endpoints of the parameter interval$[a, b]$.
- You might need additional parameters in the argument:

  plot([x(t),y(t),t=a..b],numpoints=200,color=black,scaling=constrained);

  Right-clicking on the plot gives other options.

- **color=black** draws the curves in the specified color. This is useful for printing on black and white printers.
- **numpoints=200** controls the number of points used in plotting. This is useful when your plots are not smooth enough.
- **scaling=constrained** sets equal scales for the horizontal and vertical axes.

I. Graph the following parametric equations. Increase **numpoints** for smoother graphs, as needed.

$x(t)$	$y(t)$	Range
$t^2 - 4$	$t/2$	$-3 \le t \le 3$
$\cos t$	$\sin t$	$0 \le t \le 2\pi$
$5 \cos t$	$2 \sin t$	$0 \le t \le 40$
$\cos^3 t$	$\sin^3 t$	$0 \le t \le 2\pi$
$\cos^3 2t$	$\sin^3 4t$	$0 \le t \le 2\pi$
$\cos^3 2t$	$\sin^3 3t$	$0 \le t \le 2\pi$
$\sin^3 9t$	$\cos^3 7t$	$0 \le t \le 2\pi$
$t + 3 \cos 15t$	$t + 3 \cos 16t$	$-5 \le t \le 8$

II. In class we studied the cycloid. Now, consider the hypocycloid, which is given by the equations

$$x = (\alpha - \beta) \cos t + \beta \cos(\frac{\alpha - \beta}{\beta} t) \tag{7.2}$$

$$y = (\alpha - \beta) \sin t - \beta \sin(\frac{\alpha - \beta}{\beta} t), \tag{7.3}$$

with $0 \le t \le b$. Start with $b = 2\pi$.

A possible setup for your worksheet would be

```
restart: alpha:=4: beta:=1:
x:=(alpha-beta)*cos(t)+beta*cos((alpha-beta)/beta*t);
y:=(alpha-beta)*sin(t)-beta*sin((alpha-beta)/beta*t);
plot([x,y,t=0..2*Pi]);
```

- Use the values $\beta = 1$ and $\alpha = 3, 4, 5, 6, \ldots$.
- Try $\beta = -3$ with various α's and note the effects. (e.g, $\alpha = 1, 2, 3, \ldots$.) Make sure to adjust b to capture the entire graph. What effect does changing **numpoints** have on your graphs? (Actually, for negative β's you get epicycloids!)
- Repeat the last step with $\beta = -17$.
- Play with the parameters b, α, and β to see if you can come up with some interesting plots. Make notes about the effects of various values of the parameters, such as large vs small values of **numpoints**, negative and positive values of α and β. Try to keep α and β relatively prime.

Exercises

▷ **Exercise 7.1** Write y as a function of x for the first curve in the above problems.

▷ **Exercise 7.2** In part I above you plotted $x = 5 \cos t, y = 2 \sin t$, for $0 \le t \le 40$. What did you get? How does this differ from what you expected?

▷ **Exercise 7.3** Discuss the affects of changing the parameters in the graphs of equation (1.3). Do the parameters have to be integers? For example, what would you expect for $\beta = 1$ and $\alpha = 3.5$?