Problems in Cooling and Limited Growth

Problem: A detective finds a body at 22 C in a 17 C room. The body cools to 20 C in the next hour. When was the person last alive? [Normal Body Temperature = 37 C]

Take t = 0 at the time the body was discovered. Then

$$T(t) = 17 + (22-17)e^{kt} = 17 + 5e^{kt}.$$

In one hour the body is at 20 C, so $T(1) = 17 + 5e^{k} = 20$. Therefore,

$5e^{k} = 3 \Rightarrow k = \ln\frac{3}{5} \approx -0.5108$. This is the same as we obtained in class. See how much easier it is picking this starting time. ☺

Now, we have $T(t) = 17 + 5e^{-0.5108t}$. When was the body at 37 C? Let’s see:
$37 = 17 + 5e^{-0.5108t} \Rightarrow 5e^{-0.5108t} = 20$. Solving this for *t*, we have $t = -\ln(4)/0.5108 \approx -2.7$ hrs. The negative means the body was alive at an earlier time.

Problem: 400 fish are stocked in a lake. The carrying capacity of the population is 10000. If the population triples in one year, then how long does it take for the population to reach 5000?

We set up a general function in class: $A = \frac{K - P_0}{P_0} = \frac{9600}{400} = 24$ and $P(t) = \frac{K}{1 + Ae^{kt}} = \frac{10000}{1 + 24e^{-kt}}$.

Now, in order to answer the question, we first need *k*. Use the fact that the population triples in one year. Then

$$P(1) == \frac{10000}{1 + 24e^{-k}} = 3(400) \text{ or } \frac{10000}{1 + 24e^{-k}} = 1200$$

We need to solve this for *k*. Isolate the exponential and exponentiate.

$\frac{10000}{1200} = 1 + 24e^{-k} \Rightarrow 24e^{-k} = 8.33 - 1 = 7.33$ So, $e^{-k} = \frac{7.33}{24} \Rightarrow k = 1.186$.

Our model is now, $P(t) = \frac{10000}{1 + 24e^{-1.186t}}$

When is will the population reach 5000? Set $5000 = \frac{10000}{1 + 24e^{-1.186t}}$ and solve for the time.

Rearrange: $1 + 24e^{-1.186t} = 2$ This implies $e^{-1.186t} = 1/24$. So, $t = -\ln(1/24)/1.186 \approx 2.68$ years.