Calculus Review

Differentiation

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
(cf(x))' &= cf'(x)\\
(f(x)+g(x))' &= f'(x)+g'(x)\\
(f(x)g(x))' &= f'(x)g(x)+f(x)g'(x)\\
\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}\\
\frac{d}{dx}\left(f(g(x))\right) &= f'(g(x))g'(x)
\end{aligned}
$$

Derivatives

Function	Derivative
a	0
x^n	nx^{n-1}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{1}{x}$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
$\tan ax$	$a\sec^2 ax$
$\csc ax$	$-a\csc ax\cot ax$
$\sec ax$	$a\sec ax\tan ax$
$\cot ax$	$-a\csc^2 ax$
$\sinh ax$	$a\cosh ax$
$\cosh ax$	$a\sinh ax$
$\tanh ax$	$a\,\mathrm{sech}^2\, ax$
$\mathrm{csch}\, ax$	$-a\,\mathrm{csch}\, ax\coth ax$
$\mathrm{sech}\, ax$	$-a\,\mathrm{sech}\, ax\tanh ax$
$\coth ax$	$-a\,\mathrm{csch}^2\, ax$

Integration

$$\int cf(x)\,dx = c\int f(x)\,dx$$

$$\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int_a^b f(x)\,dx = F(b)-F(a) \text{ for } f(x)=F'(x) \text{ cont. on } [a,b]$$

Integrals

Function	Indefinite Integral
a	ax
x^n	$\frac{x^{n+1}}{n+1}$
e^x	e^x
$\frac{1}{x}$	$\ln x$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert$
$\sec^2 x$	$\tan x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\mathrm{sech}^2\, x$	$\tanh x$
$\frac{1}{a+bx}$	$\frac{1}{b}\ln(a+bx)$
$\frac{1}{1+x^2}$	$\tan^{-1} x$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} x$

Hyperbolic Functions

$$
\begin{aligned}
\sinh x &= \frac{e^x-e^{-x}}{2}\\
\cosh x &= \frac{e^x+e^{-x}}{2}\\
\tanh x &= \frac{\sinh x}{\cosh x} = \frac{e^x-e^{-x}}{e^x+e^{-x}},\\
\mathrm{sech}\, x &= \frac{1}{\cosh x} = \frac{2}{e^x+e^{-x}},\\
\mathrm{csch}\, x &= \frac{1}{\sinh x} = \frac{2}{e^x-e^{-x}},\\
\coth x &= \frac{1}{\tanh x} = \frac{e^x+e^{-x}}{e^x-e^{-x}}.\\
\sinh^{-1} x &= \ln\left(x+\sqrt{1+x^2}\right),\\
\cosh^{-1} x &= \ln\left(x+\sqrt{x^2-1}\right),\\
\tanh^{-1} x &= \frac{1}{2}\ln\frac{1+x}{1-x}.
\end{aligned}
$$

Trigonometric Identities

$$
\begin{aligned}
\sin^2\theta+\cos^2\theta &= 1\\
\tan^2\theta+1 &= \sec^2\theta\\
1+\cot^2\theta &= \csc^2\theta\\
\sin(A\pm B) &= \sin A\cos B\pm\sin B\cos A,\\
\cos(A\pm B) &= \cos A\cos B\mp\sin A\sin B.\\
\sin(2A) &= 2\sin A\cos B\\
\cos(2A) &= \cos^2 A-\sin^2 A\\
\sin^2 A &= \frac{1-\cos 2A}{2},\\
\cos^2 A &= \frac{1+\cos 2A}{2}.\\
\sin A\cos B &= \frac{1}{2}(\sin(A+B)+\sin(A-B)).\\
\cos A\cos B &= \frac{1}{2}(\cos(A+B)+\cos(A-B))\\
\sin A\sin B &= \frac{1}{2}(\cos(A-B)-\cos(A+B))
\end{aligned}
$$

Special Values

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
0	1	0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{2}$	0	1	undefined

Hyperbolic Identities

$$
\begin{aligned}
\cosh^2 x-\sinh^2 x &= 1\\
\tanh^2 x+\mathrm{sech}^2\, x &= 1\\
\cosh(A\pm B) &= \cosh A\cosh B\pm\sinh A\sinh B\\
\sinh(A\pm B) &= \sinh A\cosh B\pm\sinh B\cosh A \quad (1)\\
\cosh 2x &= \cosh^2 x+\sinh^2 x\\
\sinh 2x &= 2\sinh x\cosh x\\
\cosh^2 x &= \frac{1}{2}(1+\cosh 2x)\\
\sinh^2 x &= \frac{1}{2}(\cosh 2x-1)
\end{aligned}
$$