MAT 162 Exam IV Review

1. Find the limit if it exists.
   a. $a_n = \dfrac{3n^2 - 2n + 1}{2n^2 - 1}$
   b. $a_n = \left(3n^2\right)^{1/n}$
   c. $a_n = \left(\dfrac{n}{n-2}\right)^n$
   d. $a_n = (-1)^n \dfrac{n+1}{n-1}$
   e. $a_n = \dfrac{\ln n^2}{n}$
2. List the first five terms of the sequence.
   a. $a_n = \dfrac{2n}{3n-1}$
   b. $a_{n+1} = \dfrac{1}{1+a_n}$, $a_1 = 1$.
3. Find the sum of the following:
   a. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{p}\right)^n$
   b. $\displaystyle\sum_{n=0}^{\infty}[2(0.1)^n + 0.2^n]$
   c. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$
4. Express $0.\overline{123}$ as a ratio of integers.
5. Determine if the following series converge absolutely, converge conditionally, or diverge.
   a. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{2^{2n}}$
   b. $\displaystyle\sum_{n=1}^{\infty}n^{-3/2}$
   c. $\displaystyle\sum_{n=1}^{\infty}\frac{2n^3 - 3n}{n^4 + n}$
   d. $\displaystyle\sum_{n=2}^{\infty}ne^{-2n}$
   e. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$
   f. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^n n^2}{n!}$
   g. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$

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6. Find the radius of convergence and interval of convergence of the series:

   a. $\sum_{n=1}^{\infty} nx^n$

   b. $\sum_{n=0}^{\infty} \frac{n}{4^n}(x-2)^n$

   c. $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$

7. Write the power series representation for the given function about $x = a$. Determine the interval of convergence of each when possible.

   a. $f(x) = \frac{1}{1+4x^2}$, $a = 0$.

   b. $f(x) = \frac{2}{3x+4}$, $a = 0$.

   c. $f(x) = xe^x$, $a = 1$.

   d. $f(x) = \ln x$, $a = 2$.

   e. $f(x) = \sqrt{1+x}$, $a = 0$.

8. Use series expansions to approximate the following integrals to three decimal places:

   a. $\int_0^{0.5} \sin(x^2)\,dx$.

   b. $\int_0^{0.2} e^{-x^2}\,dx$.

9. Use series to evaluate $\lim_{x\to\infty} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$.

10. Use binomial series to expand the given function as a power series.

    a. $f(x) = \sqrt{1-2x}$.

    b. $f(x) = \frac{1}{(2+x)^3}$.

11. In the expansion of $(x-2y)^{16}$, what is the numerical coefficient of $x^{11}y^5$?

12. Using an appropriate binomial series expansion, $\sqrt{8.2}$ to four decimal places.