**MAT162 Practice for Exam #2**

**1. Equations** Find the general solution to the given differential equations. When an initial condition is given, find the particular solution.

a. $\frac{dx}{dt} = -2t,\ x(0) = 4.$

b. $\frac{dy}{dx} = y(1-y).$

c. $(1+x)\frac{dy}{dx} = -3y,\ y(6) = 7.$

d. $\frac{dy}{dx} + y = x$

f. $y'' + 2y' + 2y = 0.$

g. $x'' + x' - 6x = 0,\ x(0) = 0,\ x'(0) = 5.$

h. $x' - 2x = t^2 e^{2t}.$

**2. Application Problems.**

a. A fossil contains 35 percent of the normal amount of carbon-14, whose half-life is 5580 years. What is its approximate age?

b. A bacterial population is known to have a rate of growth proportional to the population. If between noon and 2 PM the population triples, at what time should the population be 100 times the initial population?

c. A 60 gallon tank is initially half full of pure water. Brine with a salt concentration of 10 grams/gallon is pumped into the tank at a rate of 5 gallons/hour. The well-mixed contents of the tank are pumped out at 2 gallons/hour. How much salt is in the tank just as the tank fills up?

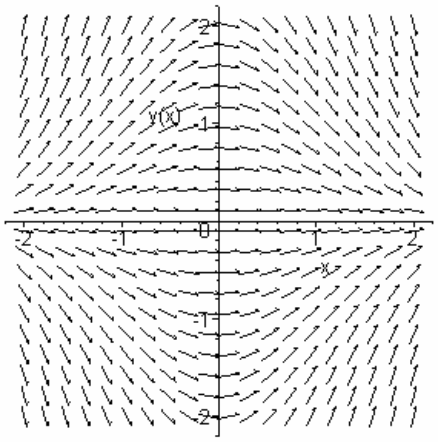
d. A population of 100 pheasants are released on an island that can only support 1,000 pheasants. Assume the growth rate for this population is 0.0025 per year. What will the pheasant population be in 5 years?

**MAT162 Practice for Exam #2**

**3. Assorted Problems**

a. Consider the problem $y' = y,\ y(0) = 1$. Use Euler's Method to approximate $y(1)$ using a step size of $h = 0.2$ and compare your value to the exact solution, $y(1)$, to find the error

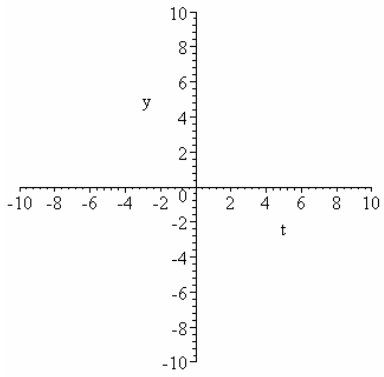
b. Sketch two different solutions on the given direction field.




c. Consider the equation $\frac{dy}{dt} = -y^2 - 2y + 24.$

i. What are the equilibrium solutions?

ii. Sketch the equilibrium solutions and typical solutions for each region of the *ty*-plane .




d. A mass of 200 g is attached to a spring and stretches it 10.0 cm. What is the spring constant? When set into motion, what is the natural angular frequency of oscillation?

e. A cup of coffee at 98 $^{o}$C is left on a table in an air-conditioned room at 20 $^{o}$C. After 5 minutes, the coffee's temperature is 38 $^{o}$C. How much longer will it take the coffee to cool down to 22 $^{o}$C?

f. You make two gallons of chili for a party. The recipe calls for two teaspoons of hot sauce per gallon, but you had accidentally put in two tablespoons per gallon. You decide to feed your guests the chili anyway. Assume that the guests take 1 cup/min of chili and you replace what was taken with beans and tomatoes without any hot sauce.
[1 gal = 16 cups and 1 Tb = 3 tsp.]
i. Write down the differential equation for the amount of hot sauce as a function of time in this mixture-type problem.

ii. What is the initial condition?
iii. Solve for this initial value problem.

iv. How long will it take to get the chili back to the recipe's concentration?