MAT 152 Review – Chapters 10-11

1. Differential Equations
    a. Types of Differential Equations
        i. $\frac{dy}{dx} = f(x)$
        ii. Separation of Variables
            1. $\frac{dy}{dt} = f(t)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(t)\,dt$
            2. General Solutions – Implicit and Explicit
            3. Initial Value Problems – Particular Solutions
        iii. Linear Differential Equations
            1. Find integrating factors and solve initial value problems
                a. $y' + P(x)y = Q(x)$

$$I(x) = \exp\int^{x} P(x)\,dx \Rightarrow (Iy)' = IQ$$

$$y(x) = \frac{1}{I(x)}\left[\int^{x} I(t)Q(t)\,dt + C\right]$$

    b. Applications
        i. Growth and Decay $\frac{dy}{dx} = ky \Rightarrow y(x) = y_0 e^{kx}$
        ii. Logistic Model $\frac{dy}{dx} = k\left(1 - \frac{y}{N}\right)y \quad \Rightarrow \quad y(x) = \frac{N}{1 + be^{-kx}}$
        iii. Lotka-Volterra – Predator-Prey Model
        iv. Mixing Problems $\frac{dy}{dt} = \text{Rate In} - \text{Rate Out}$
2. Probability
    a. Discrete Random Variables - $\{x_1, \ldots, x_n\}$
        i. Probability, $p_i = f(x_i)$ for $0 \le p_i \le 1, \sum_{i=1}^{n} p_i = 1$
        ii. Mean $m = \sum_{i=1}^{n} x_i p_i = \sum_{i=1}^{n} x_i f(x_i)$
        iii. Variance $Var(x) = \sum_{i=1}^{n} (x_i - m)^2 f(x_i) = \sum_{i=1}^{n} x_i^2 f(x_i) - m^2$
        iv. Standard Deviation $s = \sqrt{Var(x)}$
    b. Continuous Random Variables - $x \in [a, b]$
        i. Probability Density Function $f(x)$ where $\int_{a}^{n} f(x)\,dx = 1$
            1. Probability that x lies in $[c, d]$: $P(c \le x \le d) = \int_{c}^{d} f(x)\,dx$

ii. Mean $m = \int_a^b x f(x)\,dx$

iii. Variance $Var(x) = \int_a^b x^2 f(x)\,dx - m^2$

iv. Standard Deviation $s = \sqrt{Var(x)}$

c. Special Distributions

i. Uniform

1. $f(x) = \frac{1}{b-a},\ x \in [a,b]$
2. $m = \frac{a+b}{2}$
3. $s = \frac{b-a}{\sqrt{12}}$

ii. Exponential

1. $f(x) = ae^{-ax},\ x \in [0,\infty)$
2. $m = \frac{1}{a},\ s = \frac{1}{a}$

iii. Normal

1. $f(x) = \frac{1}{s\sqrt{2p}} e^{-(x-m)^2/2s},\ x \in (-\infty,\infty)$
2. $E(x) = m,\quad Var(x) = s^2$
3. Standard Normal Distribution $m = 0,\ s = 1$
4. z-scores $z = \frac{x-m}{s}$