MAT 152 Exam I Review

I. Derivatives

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b. Rules

$\frac{dc}{dx} = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\frac{d}{dx}\left(a^x\right) = (\ln a)a^x$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$
$(u+v)' = u' + v'$	$(uv)' = uv' + u'v$	$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$	$(f \circ g)' = f'[g] \cdot g'$	Chain Rule

c. Extrema

i. $f'(c) = 0$, $f''(c) > 0 \Rightarrow$ relative minimum

ii. $f'(c) = 0$, $f''(c) < 0 \Rightarrow$ relative maximum

II. Integrals

a. Rules

$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, n \neq -1$	$\int e^u \, du = e^u + C$	$\int u^{-1} \, du = \int \frac{du}{u} = \ln\lvert u \rvert + C$

b. **Fundamental Theorem of Calculus:** For any continuous function *f* on $[a,b]$ with $F'(x) = f(x)$, $\int_a^b f(x) = F(b) - F(a)$.

III. Methods of Integration

a. Substitution

Example: $\int x(x^2+5)^3 dx = \frac{1}{2}\int u^3 \, du = \frac{1}{2}\left(\frac{u^4}{4}\right) + C = \frac{1}{8}(x^2+5)^4 + C$. Here one lets $u = x^2 + 5$. Then $du = \frac{du}{dx}dx = 2x\,dx$.

Adding Limits: $\int_0^2 x(x^2+5)^3 dx = \frac{1}{2}\int_5^9 u^3 \, du = \frac{1}{2}\left(\frac{u^4}{4}\right)\Big|_5^9 = \frac{1}{8}\left(9^4 - 5^4\right)\ldots$

This works because $u = 5$ when $x = 0$ and $u = 9$ for $x = 2$.

b. Integration by Parts

$\int u\,dv = uv - \int v\,du$ and $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

IV. Applications of Integration

a. Volume of Revolution: $\int_a^b p\,[f(x)]^2 \, dx$

b. Average of a Function on $[a,b]$: $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

V. Improper Integrals

$\int_a^\infty f(x)\,dx = \lim_{b \to \infty}\int_a^b f(x)\,dx$ or $\int_{-\infty}^b f(x)\,dx = \lim_{a \to \infty}\int_a^b f(x)\,dx$