

MAT 152 Practice Problems for Exam #3

Column	Points	Score
1	12	
2	18	
3	10	
4	10	
Total	50	

Instructions:

- Do all of your work in this booklet.
- Show all of your steps** in problems for full credit.
- Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Place your **answers in a box**.
- If you need more space, you may **use the back of the page** and write **On back Page #** in the problem space.

1. (2 pts) Show that $y(x) = x^2 - 1$ is a solution of the differential equation $(y')^2 - 4y - 4 = 0$.

2. (6 pts) Classify the following equations by checking all that apply. Answer Y/N if the given equation is linear, first order, or separable. Leave no blanks.

Equation	1 st Order?	Linear?	Separable?
$y' = xy + x$			
$yy' = x + y$			
$xy'' + y = 2$			
$y' = y^2 \ln x$			

3. (4 pts) You initially have 90 g of a radioactive substance. The substance is known to decay at a rate of 5% per month.

i) Write a function $y(t)$ for the number of grams of the substance present at month t .

ii) How much is left after 10 months?

4. (18 pts) Find the general solution to the given differential equations. When an initial condition is given, find the particular solution.

a. $\frac{dy}{dx} = -2x, y(0) = 4$

b. $\frac{dy}{dt} = 100y$

c. $\frac{dy}{dx} = \frac{x}{y}$

d. $x \frac{dy}{dx} = 3y, y(2) = 5$

e. $x \frac{dy}{dx} + 2y = x$.

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5. (3 pts) In studying for this test there was a maximum amount of material M that could be learned. The rate at which the material is learned is proportional to the amount still left to be learned. Let $y(t)$ be the amount of information learned up to time t . Write down a differential equation that models studying for this text.

6. (7 pts) A 60 gallon tank is full of pure water. Brine with a salt concentration of 10 lb/gal is pumped into the tank at a rate of 5 gal/hr. The well-mixed contents of the tank are pumped out at 5 gal/hr.
i) Write the differential equation for the system.

ii) Solve the equation using the initial condition.

iii) How much salt is in the tank after 5 hours of pumping?

Bonus Solve Problem 5 for $M = 10$ and $y(0) = 0$.

7. (10 pts) A predator prey model for two species is given by the system

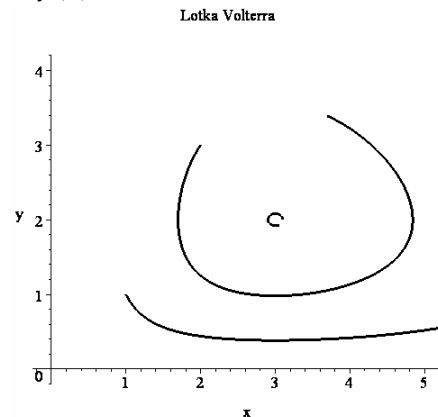
$$\begin{aligned} x' &= 2x - xy \\ y' &= -3y + xy \end{aligned}$$

a. Find $\frac{dy}{dx}$ and describe in words how one would obtain a relationship between the two populations in the form $y = y(x)$

b. Several possible solutions are shown in the plot below. The initial populations for each solution shown are

a) $x(0) = 2, y(0) = 3$. b) $x(0) = 3.1, y(0) = 2$

c) $x(0) = 1, y(0) = 1$



i) Indicate on the figure the initial populations.
ii) Describe the general behaviors of the solutions in terms of population increases and decreases.

iii) For what values are the populations constant?

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8. Consider the function $f(x) = kx^2$ on the interval $x \in [-2, 2]$.

a. Find k to make this a probability density function.

b. Find the probability $P(0 \leq x \leq 1)$.

c. Determine the mean, or expected value.

d. Find the standard deviation.

9. You roll two die numbered one to six. What is the probability that on two rolls you get two sevens?

10. The length of time in days required to learn a certain task is a random variable with probability density function $f(t) = 3e^{-3t}$.

a. Find the mean.

b. What is the probability that the time to learn the task is between the mean on one standard deviation above the mean?

11. A machine produces screws with a mean length of 2.5 cm and a standard deviation of 0.2 cm. Find the probability that a screw produced by this machine has a length greater than 2.7 cm.