**December 15, 1994** **MAT 152 FINAL EXAM** **Name:**_____

1. Let $f(x,y) = 3x^2 + xy - 5y^2$. Find

a) $\frac{\partial f}{\partial x}$.

b) $\frac{\partial^2 f}{\partial y^2}$.

c) $\frac{\partial^2 f}{\partial x \partial y}$.

2. Differentiate: $f(x) = \cos^3 4x + \ln(\tan x)$.

3.Solve for y(t): $y' = (y+1)(t+1)$.

4. The cumulative distribution function for a random variable X is $F(x) = 1 - \frac{1}{4}(2-x)^2$ for $0 \le x \le 2$.

a) Find the density function for X.

**SKIP**

b) Find Pr($1.6 \le X$).

**SKIP**

c) What is the expectation value, E(X)?

**SKIP**

5. Find the area between the curves given by $f(t) = \cos t$ and $g(t) = \sin t$ for $0 \le t \le \frac{\pi}{3}$.

(Hint: Sketch the function first to visualize problem.)

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6. Sketch the level curves for $f(x,y) = y - 2x$.

7. Do the following integrals:

a) $\int_0^1 \frac{2x}{(x^2+1)^3} dx$.

b) $\int \frac{(\ln x)^5}{x} dx$.

c) $\int x \sin 3x \, dx$.

d) $\int_1^\infty x^{-5/3} dx$.

8. Let $f(x,y) = x^2 + 4x - 2y^3 + 3y^2$. Find all of the critical points and determine if they are maxima, or minima.

9. Solve: $y' = 5 - 8y,\ y(0) = 1$.

10. Find the sum: $\sum_{k=2}^{\infty} \frac{5^k}{7^k}$.

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11. Find the second Taylor polynomial for $f(x) = (x+1)^{3/2}$. Use your result to determine f(0.1)

12. Let $f(x) = kx^2$ for $0 \le x \le 1$.
a) For what value of k is f(x) a probability density?

**SKIP**

b) Compute E(X).

**SKIP**

c) Compute Var(X).

**SKIP**

13. Let $y' = 0.4y^2(1-y)$. Sketch solutions for $y(0) = -1, y(0) = 0.1, y(0) = 2$.

14. A lottery ticket is purchased for \$10. The probability of winning \$50 is 0.08 and that of winning \$100 is 0.02. If X represents the amount won, then what are the expected earnings?

**SKIP**

15. Use the Newton-Raphson Method to compute $x = \sqrt{10}$.

**SKIP**

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16. Determine if the following series converge or diverge. Justify your answers.

a) $\sum_{k=1}^{\infty} \frac{1}{k^3}$.

**SKIP**

b) $\sum_{k=1}^{\infty} \frac{1}{3^k}$.

**SKIP**

c) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$.

**SKIP**

d) $\sum_{k=1}^{\infty} \frac{k^3}{k!}$.

**SKIP**

17. Consider and exponential distribution with and expected value of 10. Compute Pr(5≤ X).

**SKIP**

18. Two people (of the opposite sex) are stranded on an island. In five years they have eight children. Assume that the population on the island grows at a rate proportional to the square of the population.

a) Write the differential equation that the population satisfies.

b) Solve the differential equation and determine all of the constants in the problem.

c) What will the population be in six years?

19. You are stranded on an island and need to construct a rectangular canvas shelter with a back, two sides and a top. You only have 96 square feet of canvas. Explain how you would determine the dimensions of your shelter, using what you have learned in calculus and the ingenuity you brought to the island.