Special Relativity

Thursday, September 11, 2008
4:45 AM

Theory of Special Relatvity

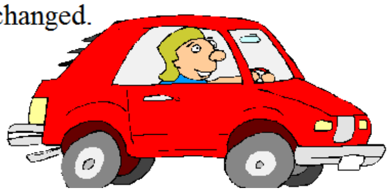
Einstein's Most Recognized Theory

From two postulates: Obervers traveling at different constant velocities observe that

1. Simultaneous events are not simultaneous to all.

2. Time and space measurements are not the same.

Our notions of space and time have changed.



1. Relative Motion
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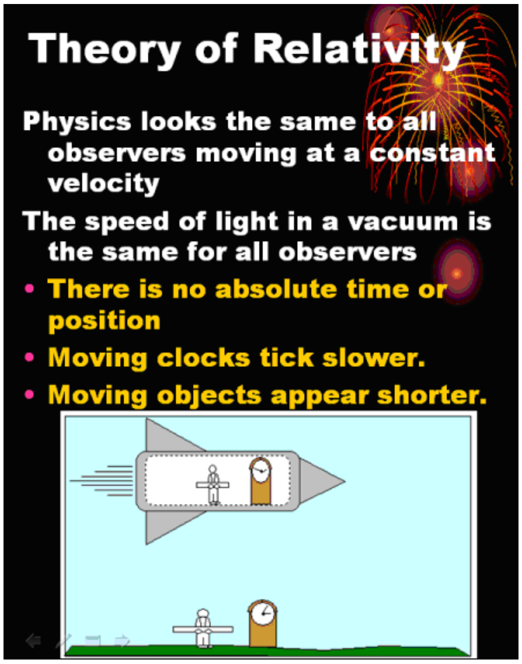


The Principle of Relativity

Galileian Relativity

"Any two observers moving at constant speed and direction with respect to one another will obtain the same results for all mechanical experiments."

Galileo's Parable of the Ship

Space and Time

Space - location given with reference to a reference point.

Time - occurance of events as measured by clocks.

Newton - There are absolute references for space and time.

Einstein - Space and Time are relative.

Dimensions

How Many Dimensions Are There?

Describe -
One Dimension
Two Dimensions
Three Dimensions ... and higher?

The Speed of Light

Recall:

Maxwell predicted electromagnetic waves in 1860
Hertz made first radio wave in 1887
Michelson and Morley could not detect the aether 1887

There were models to try and understand.

Einstein:

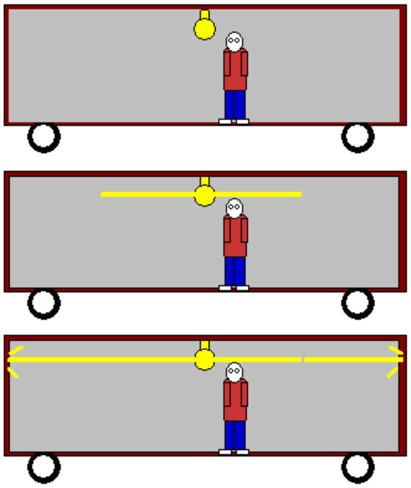
The speed of light in a vacuum is constant = ***c***.

Speed = distance/time =constant

Consequence 1 - Simultaneity

Events that are simultaneous for one observer are not necessarily simultaneous for another observer.

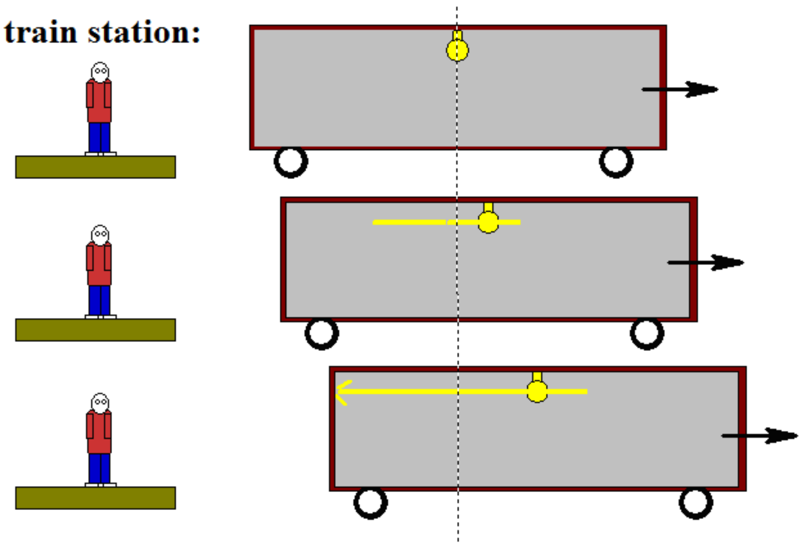
Observer on a train car:



Consequence 1 - Simultaneity

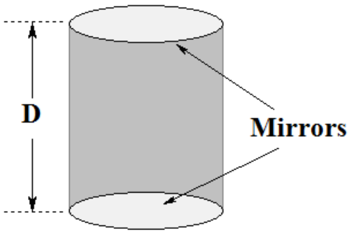
Events that are simultaneous for one observer are not necessarily simultaneous for another observer.

Observer at train station:



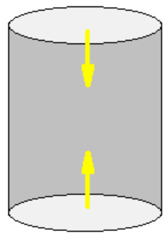
Light Clocks

We need reliable clocks to use for showing time dilation.



A light ray leaves the bottom mirror.
It strikes the top mirror and returns.

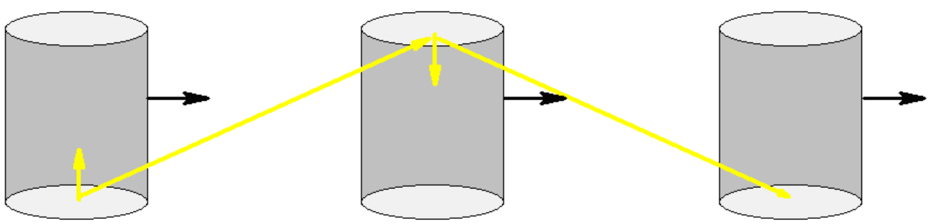
How long does it take?



Consequence 2 - Time Dilation

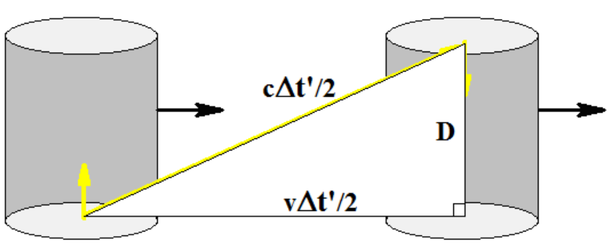
For an observer with the clock: Δt = **distance/speed = 2D/c**

For an observer watching the clock fly by at speed **v ...**



The Time Dilation Equation

We compare $\Delta t = 2D/c$ with $\Delta t'$ as measured by a stationary observer.



For stationary observer:

Clock moves distance = $v\Delta t'$ for full trip

Beam moves distance = $c\Delta t'/2$ for half trip

Geometry => $(c\Delta t/2)^2 + (v\Delta t'/2)^2 = (c\Delta t'/2)^2$

Solve for $\Delta t'$:

$$\Delta t' = \gamma \Delta t, \quad \gamma = 1/\sqrt{1 - v^2/c^2}$$

Time Dilation Example

$$\Delta t' = \gamma \Delta t, \quad \gamma = 1/\sqrt{1-v^2/c^2}$$

Moving at 250 m/s (560 mph) for 1 hour on the Earth:

$$\gamma = 1/\sqrt{1 - 250^2/300000000^2}$$
$$= 1.000000000000347...$$

So,

$\Delta t = 0.999999999999653$ hr

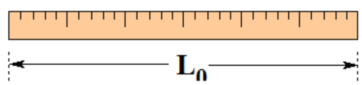
$\Delta t' - \Delta t = 0.000000000000347$ hr

$= 1.249$ ns

Consequence 3 - Length Contraction

For observer moving with the stick:

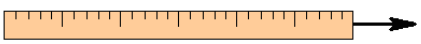
Measuring stick length = L_0



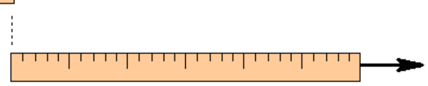
$\Delta t' = L_0/c$

For observer watching the stick passing:

Measuring stick length = L'



$\Delta t = L'/c$

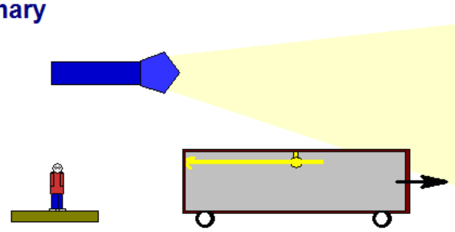


$\Delta t' = \gamma \Delta t \Rightarrow L_0/c = \gamma L'/c \Rightarrow$

$$L' = L_0/\gamma$$

Relativity Summary

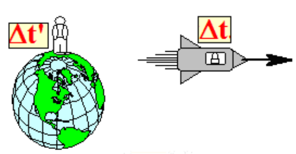
c = 3x10^8 m/s or 186,000 mi/s for everyone moving at a constant velocity.

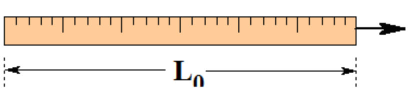


1. Events are not simultaneous for all observers.

2. Moving clocks tick slower.

3. Moving rods are shorter.

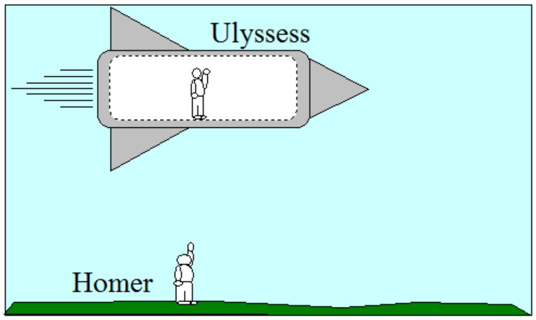




$$L' = L_0/\gamma$$

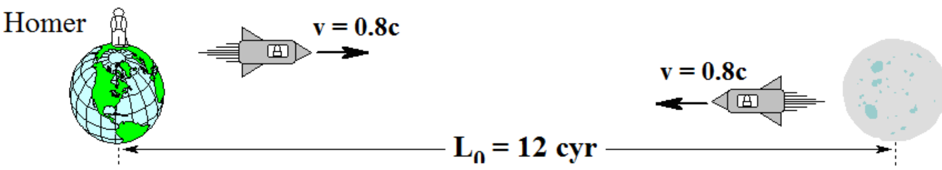
$$\Delta t' = \gamma \Delta t, \quad \gamma = 1/\sqrt{1-v^2/c^2}$$

The Twin Paradox



Ulyssess leaves for a planet 12 light years away. He travels at 0.8c and returns to find his twin, Homer, waiting for him. According to Relativity they are different ages. Who is older?

The Twin Paradox: Homer's View



On Homer's clock

How many years does it take to get to the planet?

$\Delta t = L_0/v = 12$ cyr/.8c = 15 yr

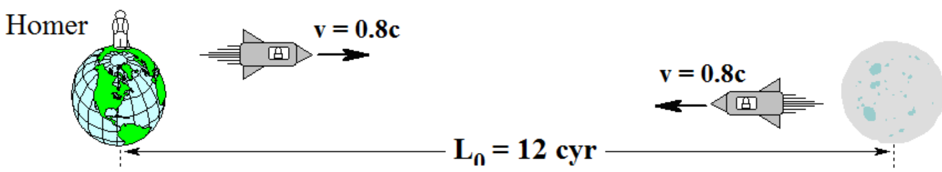
How many years for the return trip?

On Ulyssess' clock

How long compared to Homer's clock for each leg of the trip?

$\Delta t' = \gamma \Delta t \Rightarrow \Delta t = 15/(5/3) = 9$ yr

The Twin Paradox: Ulyssess' View



On Ulyssess's clock

How far is the planet?

$L = L_0/\gamma \Rightarrow L = 12/(5/3)$ cyr = 7.2 cyr

How many years does it take to get to the planet?

$\Delta t' = L/v = 7.2$ cyr/.8c = 9 yr

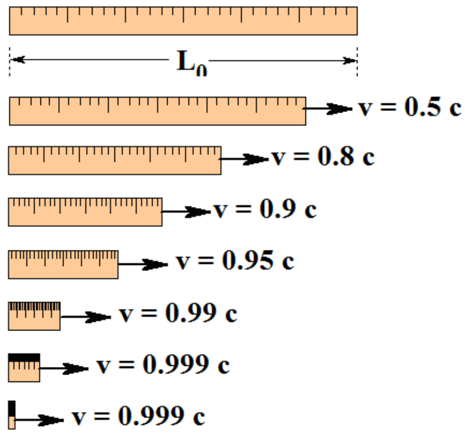
On Homer's clock

How long compared to Ulysses' clock for each leg of the trip?

$\Delta t' = \gamma \Delta t \Rightarrow \Delta t = 9/(5/3) = 5.4$ yr

The Ultimate Speed

What if the speed v = c?



Then $\gamma = 1/0!$ =>

Clocks stop, moving rods can't be seen!

$\mathbf{v = v_0 + (F/m_0)\,\Delta t'}$

$\mathbf{v = v_0 + (F/\gamma m_0)\,\gamma\Delta t}$

Thus, naively, we have
$m = \gamma m_0$ for rest mass **m_0**,

and massive objects cannot move at the speed of light!

Experimental Tests of Special Relativity

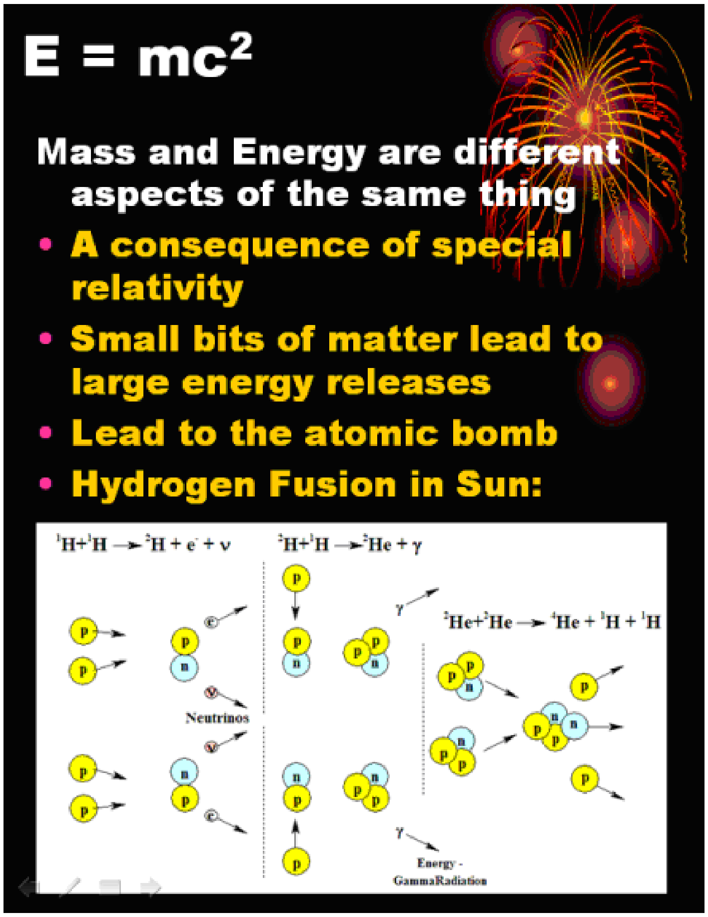
A List of Tests

Example: The lifetime of a muon. A large number of muons reach the earth after traveling thousands of meters. But, they do not live long enough according to classical theory.

Lifetime at rest = 2.6×10^{-6} s
Travels from upper atmosphere at v = 0.998c.

Without Relativity: The muon travels $x = v\,\Delta t$ = 660 m.

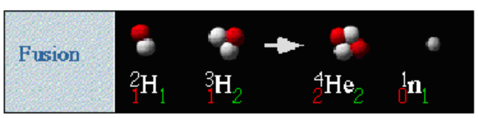
With Relativity: Observer computes $\Delta t' = \gamma\Delta t = 35\times10^{-6}$ s
So, the muon can travel 10,000 m!



Fission and Fusion

Fission - a nuclear reaction in which an atomic nucleus splits into fragments with the release of large amounts of

Fusion - combining light nuclei leading to a release of energy



Disintegration energy $Q = -m c^2$