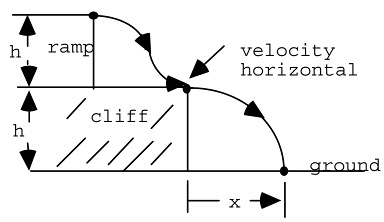
PHY 321 Practice Problems for Final

1. [1]A cannon ball of mass *m* is shot vertically. It experiences a downward force $F = - mg - kv$, where $- mg$ is the force due to gravity and $- kv$ represents air friction (v is the ball's instantaneous velocity, $k > 0$). Given that the cannonball starts at the origin of coordinates at $t = 0$ with initial upward velocity v_0, solve for the position of the particle at all subsequent times, $x(t)$.
2. A particle of mass *m* is subject to a force (one dimensional motion, $k>0$) $F(x,t) = -kx + F_0 \sin \omega t$, where $\omega^2 \neq km$. Given the initial conditions, $x(0) = x_0$, $x'(0) = v_0$, find $x(t)$ for $t > 0$.
3. A bead is released (at rest) from the top of a ramp of height *h* as shown. It moves without friction until it reaches the edge of the cliff, where it's velocity is purely horizontal. The cliff is also of height *h*. Find the distance, *x*, that the bead moves horizontally before striking the ground in terms of *h*.



4. A particle in one dimension experiences a potential, $U(x) = k\,|\,x\,|^n$, where *K* is a positive constant and $n > 1$.
    a. Find the oscillation period (as an integral) of the particle in the potential for general *n*.
    b. If the energy of the particle is increased by 2, by what factor does the period change?
5. A one dimensional potential is given by, $U(x) = -\alpha x^2 + \beta x^4$.
    a. Find the points of stable and unstable equilibrium.
    b. Find the turning points for a particle of energy $E>0$.
6. A particle is dropped in a medium for which the resistive force is given by Dv^2 with $D > 0$. Show to first order that the time taken to fall a distance *h* from rest. is given by $T \approx \sqrt{\frac{2h}{g}}\left(1 + \frac{Dh}{6m}\right)$.
7. Consider the driven, damped harmonic oscillator: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t$. Show that the instantaneous energy of the oscillator satisfies $\frac{dE}{dt} = -2\beta m x^2 + mAx\cos\omega t$.
8. A projectile is fired vertically from the Earth's surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of the Earth will it go?
9. A satellite hovers over one spot on the Earth's equator. What is the altitude of its orbit?

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[1] Some problems are adapted from http://www3.baylor.edu/Physics/open_text/classical3.html

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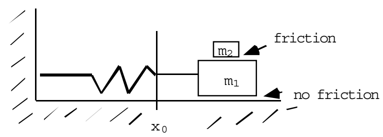
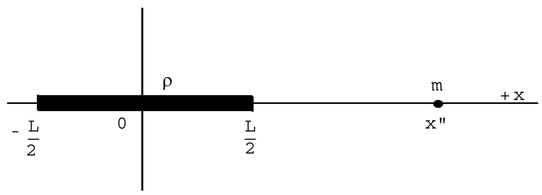
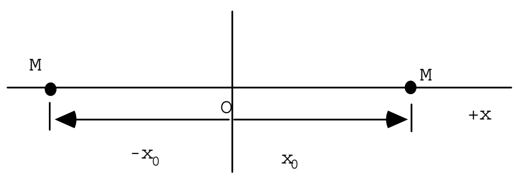
10. Consider the below system with two masses. Let x_1, x_2 be the positions of each mass relative to the equilibrium position x_0. Assume there is a frictional force proportional to velocity between the masses, but that there is no frictional force between m_1 and the ground and that the oscillations are small. Write down Newton's equations of motion for this system.



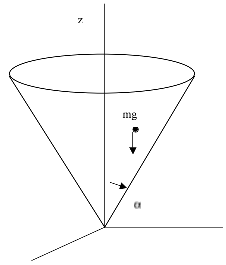
11. Find the force of gravity on a small spacecraft of mass *m*, a distance *x* from the middle of a long, straight space station (Babylon 5) of uniform density per unit length, λ, and length *L*. Take your origin of coordinates, O, in the middle of the space station, as shown.



12. Two tiny planets of equal mass, *M*, attract one another and move along the line connecting them (no rotational motion). They are initially stationary and located symmetrical distances x_0 and - x_0 from the origin, O, as shown. Show that the time necessary for the planets to crash into one another is given by $T^2 = \frac{\pi^2}{2GM} x_0^3$.



13. Consider a system of two masses in one dimension, one (m_1) attached to a wall with a spring (spring constant k_1), the other mass (m_2) attached to the first mass by another spring (spring constant k_2).
    a. Find the Lagrangian of this system.
    b. Write down Lagrange's equations of motion.
14. A small particle of mass *m* is constrained to move on the surface of a cone with opening angle α as shown. Find the Lagrangian and Lagrange's equations in some set of unconstrained variables. Show that these equations may be solved.



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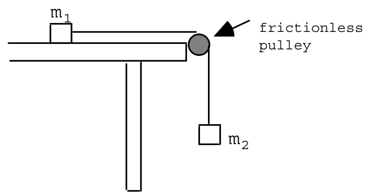
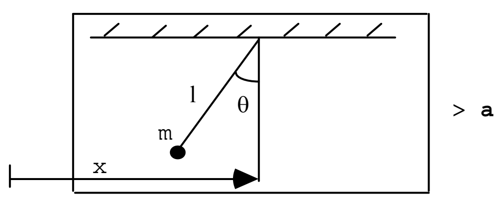
15. A particle of mass m_1 on a frictionless table is attached to another of mass m_2 by a rope. The mass m_2 is hanging off the table as shown. Formulate this as a Lagrangian constraint problem.
    a. Find the kinetic, potential energies and the Lagrangian equations of motion.
    b. Find the acceleration of mass 2 and the tension in the rope.



16. Often the KE is a quadratic function of the generalized coordinate, $T = f(q)\dot{q}^2$, where $f(q)$ is some function of the generalized coordinate, q. Assume that $U = U(q,t)$. Construct the Hamiltonian, H. Under these circumstances is H equal to the total energy? Is it a constant of the motion?
17. A particle of mass m subject to gravity is suspended from a simple pendulum of fixed length ℓ as shown. The whole system is placed in a box and accelerated with acceleration a horizontally.
    a. Choosing θ as the generalized coordinate, write down the kinetic and potential energies of the system. Show that they are explicit functions of the time.
    b. Construct the system's Hamiltonian. Is $H = T + U$? Is H a constant of the motion?



18. A mass m, attached to a string, moves on a frictionless table and the string passes through a hole in the table. Under the table a person is pulling the string to make it taut at all times. Initially the mass moves in a circle with kinetic energy E_0. The string is then slowly pulled until the radius of the circle is halved. How much work was done?
19. A mass m_1 with initial velocity V_0 strikes a mass-spring system consisting of a mass m_2 and a massless spring with spring constant k. (The spring lies between both masses.) There is no friction. What is the maximum compression of the spring?
20. A block weighing 14.0 N, which slides without friction on a 40.0° incline, is connected to the top of the incline by a massless spring of unstretched length 0.450 m and spring constant 120 N/m.
    a. How far from the top of the incline does the block stop?
    b. If the block is pulled slightly down the incline, what is the period of the resulting oscillation?

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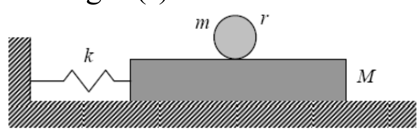
21. A lead sphere or radius *R* has a spherical hollow inside of diameter R such that the surface of the hollow passes through the center of the lead sphere and touches the right side of the sphere. The mass of the sphere before hollowing was M. With what gravitational force does the hollowed-out lead sphere attract a small spherical mass *m* at distance *d* from the center of the lead sphere?
22. A round uniform cylinder of radius r and mass m may roll, without slipping, on a horizontal surface of a massive block. The block, in turn, may move, without friction, on an immobile horizontal surface, being connected to it with a spring (see Figure below).
    a. Find the equations of motion of the system (within the plane of the picture).
    b. Find and interpret the integral(s) of motion.



23. Other Text Problems: 2.17, 3.2, 3.14, 3.44,

Other sites –
http://gallatin.physics.lsa.umich.edu/~keithr/p401/bulletinboard.html
http://www.pa.msu.edu/courses/2002fall/PHY820/
http://www-theory.lbl.gov/~horava/mtms.pdf
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