

Cosmology Review II

1. People

- a. Vesto Slipher
- b. Henrietta Leavitt
- c. Vera Rubin
- d. Kent Ford
- e. Roberts and Whitehurst
- f. E. Hubble
- g. Fritz Zwicky
- h. Sir Arthur Eddington
- i. Erwin Freundlich
- j. Allan Sandage
- k. Hipparchus
- l. Henri Poincaré

2. Objects

- a. Cepheids
- b. LMC, SMC
- c. Coma Cluster
- d. Virgo Cluster
- e. M31
- f. Supernovae
- g. White-brown-red dwarfs, neutron stars, black holes, MACHOs, WIMPs
- h. Dark halo
- i. Clusters

3. Terms

- a. Hubble time, Hubble length
- b. Horizon
- c. Planck scales
- d. Chandrasekhar limit
- e. Baryonic
- f. Deceleration parameter
- g. Standard candle
- h. Lookback time
- i. Bolometer
- j. Luminosity, flux
- k. Big Crunch, Big Freeze, Big Bounce
- l. Loitering universe
- m. Benchmark Model
- n. Density is destiny
- o. Mass to light ratio
- p. B-band filter

4. Hubble Law $v = H_0 r$, $z \approx \frac{v}{c}$, Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$

5. Hubble time H_0^{-1} , Hubble length $\frac{c}{H_0}$

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6. Steady State Model – H_0, ρ_0 constant $\Rightarrow \frac{dr}{dt} = H_0 r$

7. Horizon distance $d_p(t) = c \int_0^{t_0} \frac{dt}{a(t)}$

8. Red shift $1+z = \frac{1}{a(t_e)}, a(t_0) = 1.$

9. Friedmann Equation

a. $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \epsilon - \frac{\kappa c^2}{R_0^2 a^2}$

b. $H_0^2 = \frac{8\pi G}{3c^2} \epsilon_0 - \frac{\kappa c^2}{R_0^2},$

10. Energy density

a. $\epsilon_w = \epsilon_{w,0} a^{-3(1+w)} \Rightarrow \epsilon_m, \epsilon_r$

b. $\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_w \epsilon_{w,0} a^{-1-3w} - \frac{\kappa c^2}{R_0^2}$

11. Density parameters

a. $\Omega_{CMB} = 5.0 \times 10^{-5}, \Omega_\nu = 0.681 \Omega_{CMB}, \Omega_r = 8.4 \times 10^{-5}$

b. $\Omega_m = 0.3, \Omega_\Lambda = 0.7$ (Benchmark)

12. Models

a. Curvature Only (Zero, positive negative curvature – what happens? $a = ?, t_0 = ?, t_e = ?$)

b. Solve for $d_p(t_0) = c \int_{t_e}^{t_0} \frac{dt}{a(t)}, d_p(t_e) = \frac{d_p(t_0)}{1+z}$

c. Spatially Flat Universes

i. $a(t) = \left(\frac{t}{t_e}\right)^{2/(3+3w)}, t_0 = \frac{2}{3(1+w)} H_0^{-1}, \epsilon(t) = \frac{3c^2}{8\pi G} H_0^2 \left(\frac{t}{t_0}\right)^{-2}$

ii. Lambda Only

1. $\dot{a}^2 = \frac{8\pi G \epsilon_\Lambda}{3c^2} a^2 \Rightarrow \dot{a} = H_0 a \Rightarrow a(t) = e^{H_0(t-t_0)}$

d. Multiple-Component Universes

i. $\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2} (1 - \Omega_0),$

ii. $\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2}$

iii. $H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + 1 - \Omega_0}}$

iv. Determining eras of dominant components (Epochs), $a_m, a_{m\Lambda}, t_{crunch},$ etc.

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13. Cosmological Parameters

- a. $a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2$
- b. $q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$
- c. Look back time $t_0 - t_e \approx H_0^{-1} \left[z - \left(1 + \frac{q_0}{2}\right) z^2 \right]$

14. Luminosity

- a. flux $f = \frac{L}{4\pi S_\kappa^2(r)(1+z)^2}$
- b. Luminosity distance $d_L = S_\kappa^2(r)(1+z)$, $d_L = d(t_0)(1+z)$, [$\kappa = 0$]
- c. Parallax - $d_{pc} = \frac{1}{\theta_{arcsec}}$
- d. Angular diameter distance $d_A = \frac{\ell}{\delta\theta}$, $d_L = \frac{d_L}{(1+z)^2} = \frac{d_p(t_0)}{(1+z)}$, [$\kappa = 0$]
 - i. Standard Candles – Cepheids, Supernovae Type Ia - $L = 4 \times 10^9 L_{sun}$
- e. $d_L \propto \bar{f}$
- f. Apparent magnitude $m = -2.5 \log_{10} \frac{f}{f_x}$
- g. Absolute magnitude $M = -2.5 \log_{10} \frac{L}{L_x} = m - 5 \log_{10} \frac{d_L}{1Mpc} - 25$

15. Dark Matter

- a. What is it?
- b. Density parameters for stars (0.004), baryonic matter (0.04 ± 0.01), galaxies (0.04-0.16), clusters (0.2)
- c. Orbital speeds in spiral galaxies $v = \sqrt{\frac{GM}{R}}$
- d. Surface brightness $I(R) = I(0)e^{-R/R_s}$ ($R_s = 4$ kpc Earth, 6 kpc M31)
- e. Axis ratio $\frac{b}{a} = \cos i$
- f. Half-mass radius

16. Gravitational lensing

- a. Deflection angle $\alpha = \frac{4GM}{c^2 b}$, Sun – 1.7 arcsec
- b. Einstein radius $\theta_E = \left(\frac{4GM}{c^2 d} \frac{1-x}{x} \right)^{1/2}$
- c. Lensing event - $\Delta t = \frac{d\theta_E}{2v}$