Cosmology Review I

1. People
    a. Olbers Paradox (1826)
    b. Riemann, Lobachevski
    c. Einstein
    d. Planck
    e. Slipher
    f. Hubble
    g. Bondi, Gold, Hoyle – Steady State Model
    h. Penzias and Wilson (1965)
    i. Friedmann, Lemaitre, Robertson, Walker
2. Units – AU, pc, Mpc, Planck units
3. Olbers Paradox (1826)
4. Isotropic, Homogeneous
5. Redshift $z = \frac{\lambda_o - \lambda_e}{\lambda_e}$
6. Hubble Law $v = H_0 r,\ z \approx \frac{v}{c}$, Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$
7. Hubble time H_0^{-1}, Hubble length $\frac{c}{H_0}$
8. Steady State Model – H_0, ρ_0 constant => $\frac{dr}{dt} = H_0 r$
9. Blackbody radiation - $\varepsilon_\gamma = \alpha T^4,\ n_\gamma = \beta T^3$
10. CMB $2.725 \pm 0.001\,\mathrm{K}$
11. Equivalence Principle
12. Curvature $\alpha + \beta + \gamma = \pi + \frac{\kappa A}{R_0^2}$
13. Line elements
    a. Minkowski metric
        i. Lorentz transformation
        ii. $dx' = \gamma(dx - vdt),\ cdt' = \gamma(cdt - \beta dx),\ \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},\ \beta = \frac{v}{c}$
        iii. Time dilation, Length Contraction
        iv. Light cones, past cone, future cone, null cone
    b. $ds^2 = dr^2 + S_k(r)^2 d\Omega^2$
    c. $ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_c}{r}\right)c^2 dt^2 + \left(1 - \frac{r_c}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,\ r_c = \frac{2GM}{c^2}$
    d. Geodesics

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e. Gravitational redshift $\Delta\tau_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta\tau_A, \Phi = gh, \Phi = -\frac{GM}{R}$

Signals received at lower potential are received at faster rate.

14. Proper distance $d_p(t) = a(t)r$

15. Red shift $1 + z = \frac{1}{a(t_e)}, a(t_0) = 1.$

16. Friedmann Equation $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$

a. $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}, \frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(1 - \Omega_0),$

17. Equations of State

a. $P = \omega\varepsilon, \omega = \begin{cases} 0, & \text{nonrelativisitc matter} \\ 1/3, & \text{radiation} \\ -1, & \text{exotic, } \Lambda \end{cases}$

18. Fluid Equation $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$

19. Acceleration Equation $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$

20. Cosmological Constant $\varepsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda = -P_\Lambda$

a. $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3},$

b. $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0,$

c. $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$