Review Topics for MAT 515 Midterm

I. Basic Computations
   a. Complex Numbers
      i. Basic Operations
      ii. Know how to use polar forms

$$z = re^{i\theta},\ x = r\cos\theta,\ y = r\sin\theta \text{ and } r = \sqrt{x^2+y^2}, \tan\theta = \frac{y}{x}$$

      iii. $e^{i\pi} = -1, e^{2\pi ik} = 1$ for k an integer
      iv. Complex Modulus and complex conjugate
      v. *n*th roots and roots of unity

$$z^{1/n} = r^{1/n}(\cos\frac{\theta+2k\pi}{n} + i\sin\frac{\theta+2k\pi}{n}) \text{ for } k = 0,1,\ldots,n-1$$

      vi. Stereographic Projection (
   b. Complex Functions
      i. Determine real and imaginary parts of functions: $f(z) = u(x,y) + iv(x,y)$
      ii. Map points, lines, circles, curves, regions under a given function.
      iii. Multivaluedness, principal values, etc: logarithms and other transcendental functions, branch points.
      iv. Linear fractional transformations – basic ideas and determination given three points in each plane, cross ratio, $\frac{w-w_1}{w-w_2} : \frac{w_3-w_1}{w_3-w_2} = \frac{z-z_1}{z-z_2} : \frac{z_3-z_1}{z_3-z_2}$.
      v. Specific functions: $\frac{1}{z}, az+c, (z-a)^n, P_n(z), \frac{P_m(z)}{Q_n(z)}, e^z, \cos z, \cosh z, ..., z^{1/n}, \ln(z)$

$$\frac{1}{2}\left(z+\frac{1}{z}\right), \frac{az+b}{cz+d}, ad-bc \neq 0.$$

   c. Differentiation
      i. Compute Derivative $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$
      ii. Differentiability and CR Equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$
      iii. Harmonic Functions, $\nabla^2 u = 0$, and harmonic conjugates.
      iv. Holomorphic, Analytic, Entire, Meromorphic.

II. Basic Analysis Topics
   a. Complex Numbers - structure as ordered pairs under addition and multiplication [$z_1 z_2 = (x_1x_2 - y_1y_2, x_1y_2 + x_2y_1)$ for $z_i = (x_i, y_i), i = 1,2.$] including proofs related to commutativity, associativity, inverses, identities, etc.
   b. Complex Numbers – structure as a vector space with scalar multiplication
   c. Inequality arguments, like triangle inequality.
   d. Terminology of point sets and regions in the plane, such as limit points, boundary points, open, closed, compact, connected, Bolzano-Weierstrass, Hiene-Borel, domain
   e. Sequences and series, convergence, convergence tests
   f. Basic ideas about Stereographic projection (labels), extended complex plane, point at infinity. $\xi = \frac{2x}{x^2+y^2+1}, \eta = \frac{2y}{x^2+y^2+1}, \zeta = \xi = \frac{x^2+y^2-1}{x^2+y^2+1},\ x = \frac{\xi}{1-\zeta}, x = \frac{\eta}{1-\zeta}.$

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g. Definition of curves – simple Jordan arc, simple smooth arc, simple closed Jordan curve, Jordan Curve Theorem
h. Continuity and differentiability
i. Sufficient vs necessary CR conditions for differentiability
j. Conformal mappings, meaning of $\mathrm{Arg}(f'(z_0)), |f'(z_0)|$