Complex Variables Midterm Sample **Name _____**

1. Do the following:
   a. Convert to polar form: $5-12i$

   b. Evaluate and plot the roots of $\sqrt[3]{i-1}$.

   c. Find all values of z such that $e^z = -3$.

   d. Find the real and imaginary parts of $\cos(z-\frac{\pi}{2})$ in the simplest form.

2. Sketch the set of points $z \in \mathbb{C}$ determined by the conditions:
   a. $z = 3+i+2e^{i\theta}, 0 \le \theta \le \pi.$

   b. $|z-1| = |z+i|$

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3. Describe in detail the image of the circle $|z-i|=2$ under the inversion $w=\frac{1}{z}$.

4. (6 pts) Consider the function $u(x,y)=x^2-x-y^2$
   a. Find the harmonic conjugate $v(x,y)$

   b. For these functions determine the analytic $f(z)=u(x,y)+iv(x,y)$. (Your answer has to be written as a function of z.)

5. Let $f(z)$ be analytic in the disk $|z-1|<1$ and $f'(z)=\frac{1}{z}$. Show that $f(z)=\log z$ given that $f(1)=0$. [Why isn't this obvious?]

6. Use the triangle inequality to prove that $|z_1-z_2|\geq|z_1|-|z_2|$ for any two complex numbers.

7. Show that the function $f(z)=\bar{z}^2+z$ is not analytic.

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8. (10 pts) Define the following:
   a. Entire function

   b. Harmonic function

   c. Conformal map

   d. Jordan curve theorem

   e. Linear fractional transformation

9. Consider the collection of ordered pairs of real numbers (x, y) satisfying the addition and multiplication formulae for the complex number system. Prove the cancellation law for multiplication: If $z_1 z_2 = z_1 z_3$ and $z_1 \neq 0$, then $z_2 = z_3$.

10. Prove $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$.

11. Consider the usual stereographic projection. Locate the images of following on the sphere:
    a. $3 + 4i$.
    b. $|z| > 1$.
    c. $\operatorname{Re} z > 0$.
    d. Family of parallel lines.

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12. Find all of the zeros and poles, including their order:

$$\frac{z^5+6iz^4-11z^3-2iz^2-2z^4-12iz^3+24z^2+16iz-12z-8i}{z^2(z-3)}$$

13. Find all complex solutions of $\sin(z)=4$.

14. Determine the convergence of

a. $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n!}$.

b. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(z-3)^n$.

15. Find the image of $|z|=2$ under $f(z)=z+\frac{1}{z}$.

16. Find a linear fractional transformation that takes the points 0, 1, i into the points -2, 0, 2.

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