Course Syllabus - Nonlinear Dynamics and Chaos

Course Content

Required Texts: *Chaos: An Introduction to Dynamical Systems*, by Alligood, Sauer and Yorke, 2000.

We will cover the material in Chapters 1-9 of the text. Additional topics may be covered as time permits. You will also be assigned additional readings from other sources and projects during this course. It is the intent that this course will be fun, interesting and shed light on some current topics in applied mathematics.

What is Chaos?

Many simple nonlinear deterministic systems can behave in an apparently unpredictable and chaotic manner. Chaos was not fully appreciated until the widespread availability of digital computers for numerical simulations and the demonstration of chaos in various physical systems. The study of chaotic systems has undergone explosive growth in the past two decades. The ideas of chaos have been very fruitful in such diverse disciplines as biology, economics, chemistry, engineering, fluid mechanics, and physics.

A dynamical system is one whose evolution is deterministic: its future motion is determined by its current state and its past history. A chaotic dynamical system is characterized principally by its sensitivity to initial conditions. A well known example of this is the weather. Small changes in weather patterns can result in large changes later on.

Historically, Henri Poincaré (1854-1912) was instrumental in providing the fundamental notions of dynamical chaos as can be found at the University of Maryland Chaos site: http://www-chaos.umd.edu/misc/poincare.html:

> The idea of dynamical chaos was first glimpsed by Poincaré when he entered a contest sponsored by the king of Sweden. One of the questions in this contest was to show rigorously that the solar system as modeled by Newton's equations is dynamically stable. The question was nothing more than a generalization of the famous three body problem, which was considered one of the most difficult problems in mathematical physics. In essence, the three body problem consists of nine simultaneous differential equations (all linear, each of second order). The difficulty was in showing that the general solution converges since any solution will be given in terms of infinite series. While Poincaré did not succeed in giving a complete solution, he made such a major headway in attacking the problem that he was awarded the prize anyway. The distinguished Weierstrass, who was one of the judges, said, "this work cannot indeed be considered as furnishing the complete solution of the question proposed, but that it is nevertheless of such importance that its publication will inaugurate a new era in the history of celestial mechanics."

> To show how visionary Poincaré was, it is perhaps best if he described the Hallmark of Chaos - sensitive dependence on initial conditions - in his own words:

> *If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.* - in a 1903 essay "Science and Method"
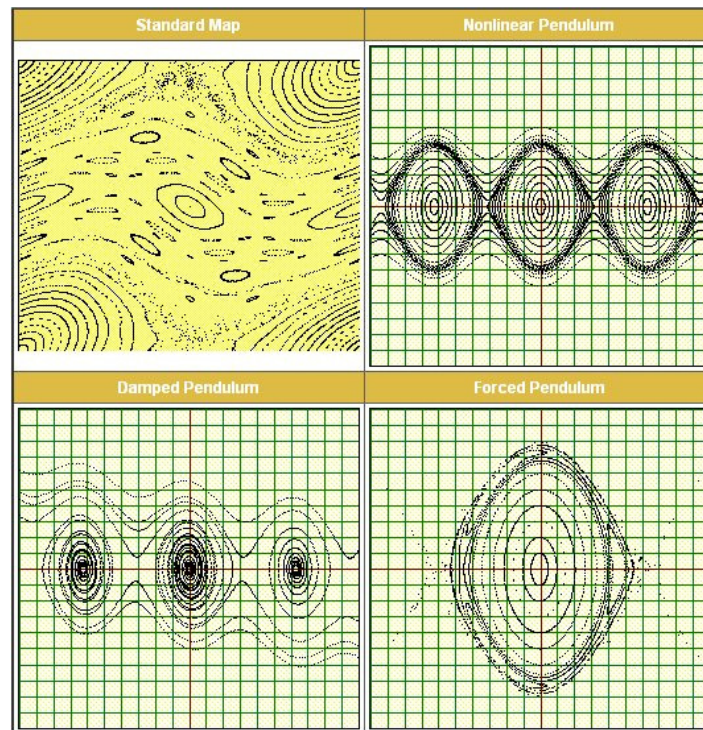
Student Learning Outcomes

Upon completion of this course, students will be able to

- Find fixed points and periodic orbits of iterated one-dimensional mappings and discuss their stability;
- Use cobweb diagrams to illustrate stability of a fixed point or periodic orbit;
- Identify commonly encountered local and global bifurcations;
- Connect Lyapunov exponents to chaos and compute them for simple examples;
- Construct and analyze a wide range of fractals;
- Understand and compute various fractal dimensions;
- Find and identify features of Mandelbrot and Julia sets;
- Understand and create Iterated Function Systems;
- Determine fixed points, the linearization about the fixed points, and their stability for one- and two-dimensional systems of ODEs;
- Understand the distinction between hyperbolic and non-hyperbolic fixed points;
- Sketch a phase portraits of linear and nonlinear one- and two-dimensional systems of ODEs;
- How to identify features of nonlinear systems such as periodicity, limit cycles, and strange attractors;
- Use the Poincaré-Bendixson theorem and polar coordinates to investigate the existence of limit cycles;
- Investigate bifurcations of one-dimensional dynamical systems in general; draw bifurcation diagrams;
- Apply standard nonlinear techniques in applications from different disciplines;

Group Work

In this course you will occasionally work with other students to complete a task. For many of you group work will be a new experience. In order to make this experience both productive and enjoyable, we offer the following suggestions:

- Start the project as soon as it is assigned. Do not put it off until the last minute. Some of the assignments will take time and working in a group may require more time due to scheduling difficulties. Read over the entire assignment, carefully before discussing or completing any part of it.

- Initially, you may have no idea as to how to get started. Don't panic! Discuss the project with the group and generate some ideas. Assign tasks to each member (leader, recorder, computation, analysis, author, editor.)

- Projects are not always as straightforward as standard homework assignments. You may need to make some assumptions and later justify these assumptions, indicating how they affect your results.

- The final report should be thoughtful, well-written and neatly organized. It should summarize your approach to the problem, present your results and conclusions, and be furnished with full explanations.

- If you have investigated the project as far as possible and still have questions, or there is a need for clarification of some point, then discuss them with your instructor before writing the report.

Web Pages/Email:

This syllabus as well as a variety of other relevant information for this class will be posted on the internet. You are encouraged to log onto this page at least weekly and send email to your instructor. You can also email the instructor for hints to homework questions, after working on them. (I will try to get back to you with an answer within a day. Sometimes the hints may be posted on the web page.)

You will find other useful materials, such as links to tutorials, sample problems, etc. provided by people at other universities. Those proficient with the web may explore and find other sites, which we can add to our resources. So, watch for additions, changes, and announcements for the class.

Course Requirements:

Graduate/Undergraduate Student Requirements: This course has students of different levels (undergraduate and graduate students). The lectures will be the same for both classes. There will be some differences in expectations for the homework assignments and exams. These are listed in the appropriate sections of the website. Also, the graduate students are expected to read more advanced materials and make more in-depth and higher quality contributions in group work and their research papers.

Homework: Homework assignments will be assigned periodically and you will be told when the work is due. There will be a penalty of 5% for each class that it is late**.**

Project: Several projects will be assigned during the course. A major project will be carried out during the second half of the semester. This will be a group effort, but the individual student grade will be based upon the efforts, accuracy, promptness exhibited in carrying out their part of the project. The end result will be a webpage and a 10-15 minute presentation during the allotted final time: **May 2, 3:00-6:00 P.M.**

Exams and Grades:
There will be two exams for this course. The material and the tentative dates for the exams are given below:

Exam	Chapters	Date
1	Ch 1-3	Feb 15
2	Ch 4-7, 13	Apr 5

Your final grade will be based on the following:

Two Exams	60%
Homework	20%
Projects	20%

90-100	A
80-89.5	B
70-79.5	C

In some cases borderline grades may be modified by a plus, or a minus, if the instructor determines that such grades are earned. Undergraduates can receive a D (60-69.5) but graduate students cannot receive a D.

Academic Honor Code:
All members of UNCW's community are expected to follow the academic Honor Code. Please read the UNCW Honor Code carefully (as covered in the UNCW Student Handbook). Academic dishonesty in any form will not be tolerated in this class. Please be especially familiar with UNCW's position on plagiarism as outlined in the UNCW Student Handbook. Plagiarism is a form of academic dishonesty in which you take someone else's ideas and represent them as your own.

Student Disabilities: UNCW Disability Services supplies information about disability law, documentation procedures and accommodations that can be found at http://uncw.edu/disability/. To obtain accommodations the student should first contact Disability Services and present their documentation to the coordinator for review and verification.

Campus Respect Compact. UNCW has recently instituted a Respect Compact to affirm our commitment to a civil community, characterized by mutual respect. That Compact will soon be affixed to the wall of each classroom and can be accessed at:
http://uncw.edu/diversity/documents/ApprovedSeahawkRespectCompact8x10.08.09.pdf

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