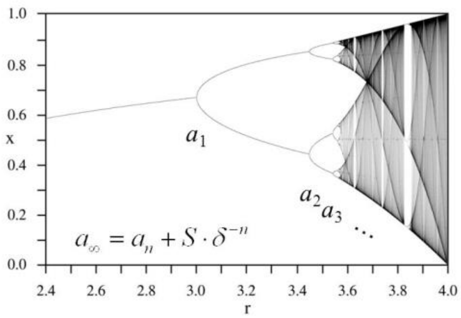
Introduction to Dynamical Systems and Chaos – Spring 2018

MAT 475 & MAT 564

Dr. R. Herman

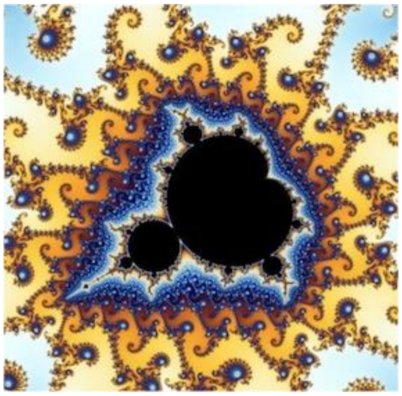
Prerequisites: Linear algebra and an interest in learning about the nonlinear mathematics; exposure to differential equations is recommended.



Feigenbaum number: $\delta = 4.6692...$

litchell Feigenbaum (July 1978) "Quantitative universality for a class of nonlinear 'ansformations," *Journal of Statistical Physics*, vol. 19, no. 1, pages 25–52.

Nonlinear Dynamical Systems and Chaos. Study of nonlinear discrete and continuous dynamical systems. Topics include the iteration of one and two dimensional maps; chaos and fractals in one and two dimensions; continuous time systems and difference equations; stability analysis of fixed and periodic points; bifurcations theory; dissipative flows and attractors; measures of chaotic time series; and, select advanced topics.

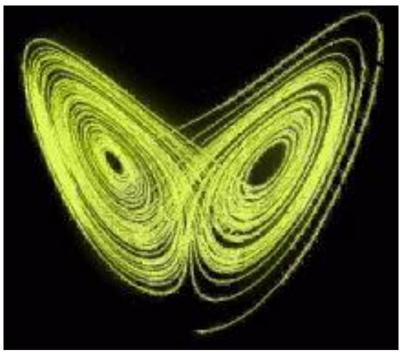


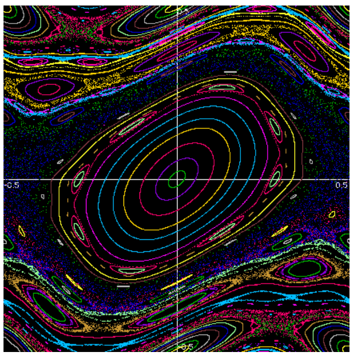
What is Chaos? What are Fractals?

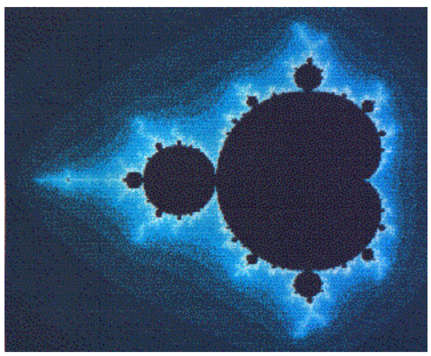
Many simple nonlinear deterministic systems can behave in an apparently unpredictable and chaotic manner or they can exhibit intriguing repetitions at finer scales. Dynamical systems can be as complicated as the weather or as simple as the perfect card shuffle. We will explore the mathematics and computer simulations to characterize chaotic behaviors.

A chaotic dynamical system is characterized by its sensitivity to initial conditions. A well-known example of this is the weather. Small changes in weather patterns can result in large changes later on. How does one identify and quantity chaotic dynamics? Simple deterministic systems can be chaotic.

Fractals are characterized by self-similarity, patterns that repeat over and over again on smaller scales. Fractals have fractional dimension. Mandelbrot used them to model roughness in nature, such as the capillary networks in the lungs.







"If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon." – Henri Poincaré, 1903

More information about chaos can be found at **http://people.uncw.edu/hermanr/chaos**