Nonlinear Dynamics Exam I **Name _____**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space, or use the **attached blank sheet**.

Try to answer as many problems as possible. Provide as much information as possible, such as sketches, etc. Show all work for full credit. **Do your best and pace yourself**!

Page	Pts	Score
1	**16**	
2	**16**	
3	**18**	
4	**16**	
5	**14**	
Total	**80**	

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1. (16 pts) Define the following terms:
    a. Eventually periodic.

    b. Saddle points for maps on R^n.

    c. Hyperbolic points.

    d. Chaotic point.

    e. Butterfly effect.

    f. Bifurcation diagram.

    g. Lyapunov exponent.

    h. Conjugacy

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2. (10 pts) Consider the map $g(x) = 3.05x(1-x)$.
    a. Find the fixed points and determine their stability.

    b. Find the period two points and determine their stability. [If you cannot solve for the periodic orbits, at least describe the process. You should know if period two orbits are stable or unstable for the given value of r.]

3. (6 pts) How many period twelve orbits does the tent map have? (Show your work.)

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4. (8 pts) Consider the linear map in the plane: $\mathbf{f}(x, y) = (5x + 2y - 2, 3x + 4y - 3)$
   a. Find the fixed point(s).

   b. Classify the behavior of points near the fixed point(s).

5. (10 pts) Consider the nonlinear map: $\mathbf{f}(x, y) = (0.5x + 2xy, y - x^2)$
   a. Find the fixed point(s)

   b. When possible, describe the behavior of points near the fixed point(s).

6. (5 pts) Under the tent map, what is the itinerary for the orbit starting at $x_1 = \frac{1}{6}$? What term most accurately describes the resulting orbit: periodic, eventually periodic, asymptotically periodic, chaotic? _____

7. (6 pts) Write the following in the given base:
   a. $\frac{1}{6}$ in base 2.

   b. $0.10\overline{100}_2$ in base 10

8. (5 pts) Prove that $G(C(x)) = C(T(x))$ for $\frac{1}{2} \le x \le 1$. Recall that $C(x) = \frac{1}{2}(1 - \cos \pi x)$.

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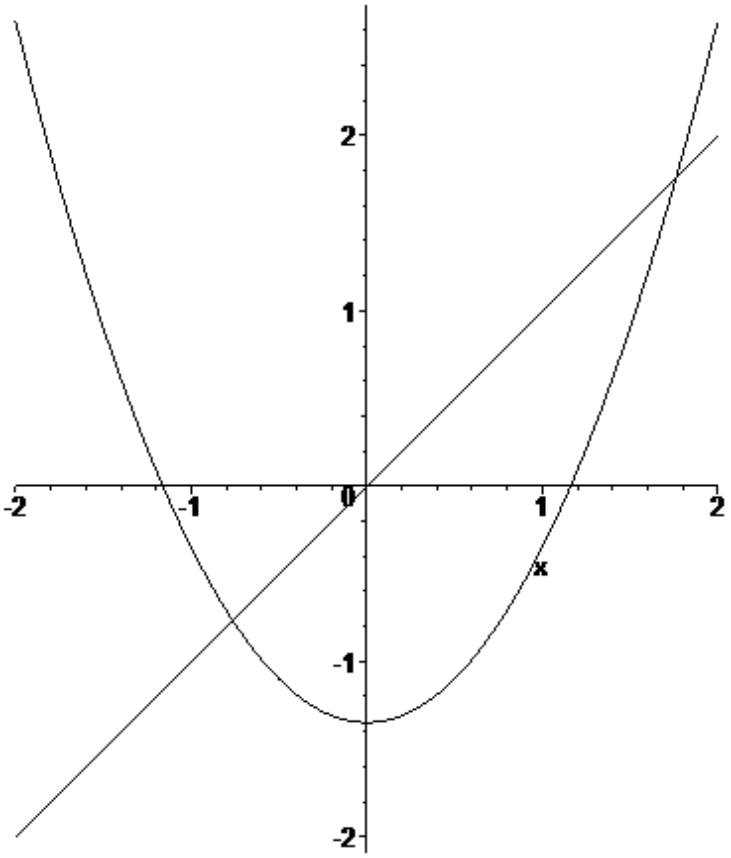
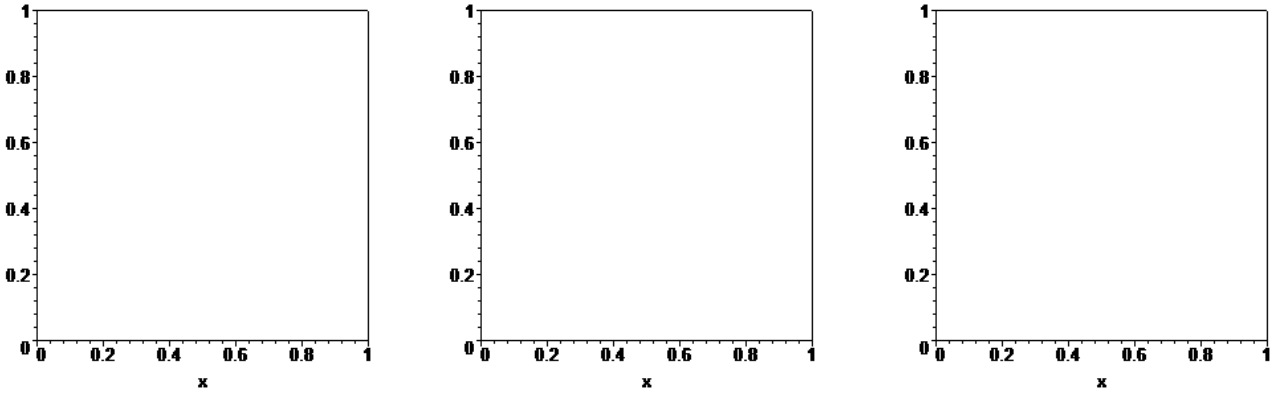
9. (5 pts) Carefully draw a cobweb diagram for the initial condition $x_0 = 1$. Describe the behavior of the orbit.



10. (9 pts) Consider the map T on [0,1] given by $T(x) = \begin{cases} \frac{3}{2}x, & 0 \le x \le \frac{1}{2}, \\ \frac{3}{2}(1-x), & \frac{1}{2} \le x \le 1. \end{cases}$

a. Sketch T, T^2, and T^3
b. How many points have
Period 1 (fixed point) _____, Period 2 _____, or 3 _____?

$T(x)$ $T^2(x)$ $T^3(x)$



Extra Space – If you use this page, please keep it organized and note in the problem space that you have work on this page.