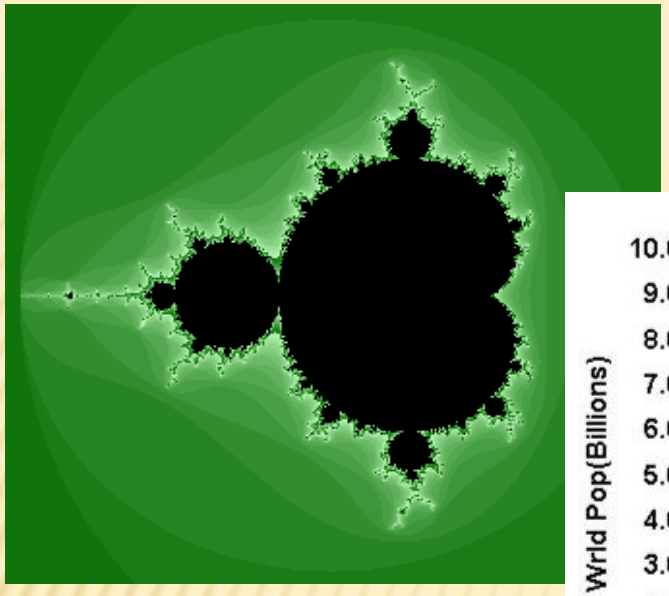
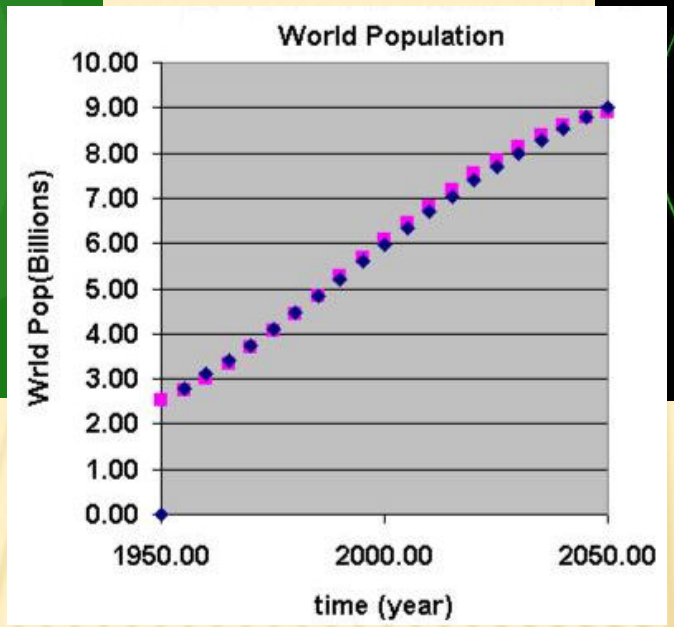
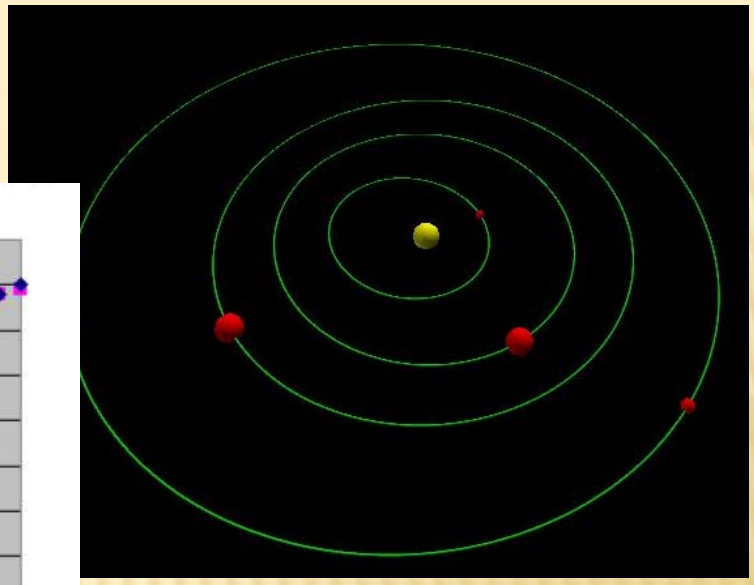






Dr. Russell Herman

Mr. David Glasier

MATHEMATICAL MODELING

SUMMER VENTURES IN SCIENCE AND MATHEMATICS 2008

COURSE ABSTRACT

This course will be an introduction to the mathematical modeling of systems in nature. Electronic data acquisition and computer analysis of experiments may be incorporated. Specific topics shall include an introduction to mathematical model building, exploring simple hands-on and simulated experiments using microcomputers mathematical software. Students will have the opportunity to develop and analyze mathematical models of real world continuous and discrete processes.

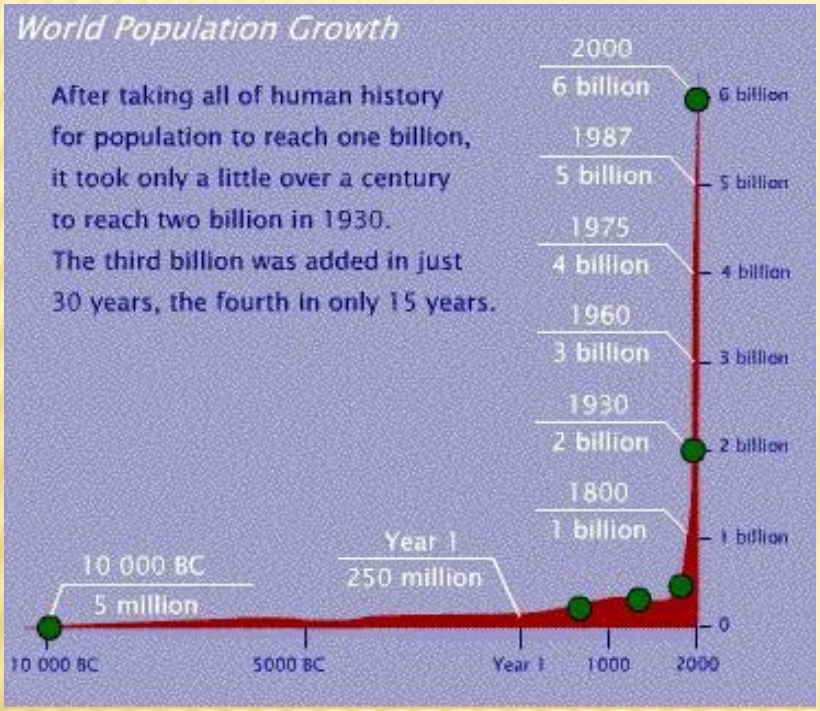
WHAT IS MATHEMATICAL MODELING?

WHAT IS MATHEMATICAL MODELING?

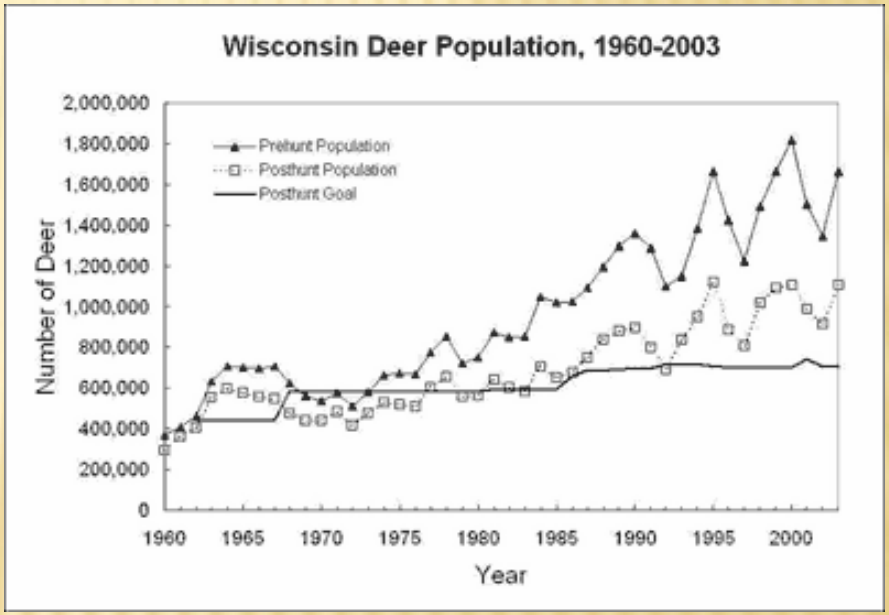
- the process of representing a real-world phenomenon as a set of mathematical equations
- Fields
  - Natural sciences, engineering, social sciences
- Classifications
  - Linear vs nonlinear
  - Deterministic vs stochastic
  - Continuous vs discrete

EXAMPLES OF MODELS

POPULATION MODELS



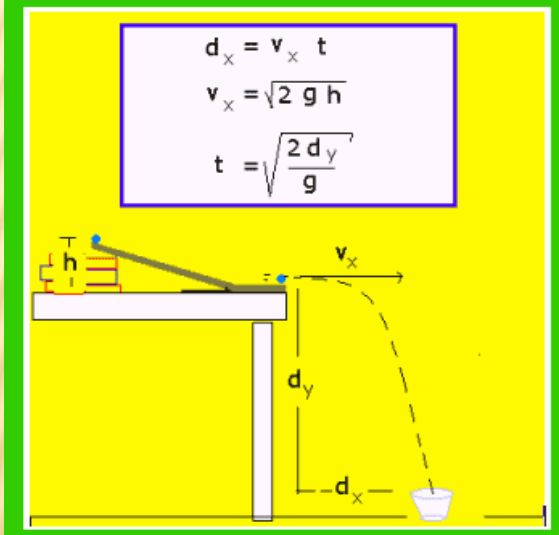
http://www.sustainablescale.org/AreasofConcern/Population/PopulationandScale/QuickFacts.aspx



http://council.wisconsinforestry.org/deer/deerpop.php

PHYSICAL MODELS

Projectile Motion



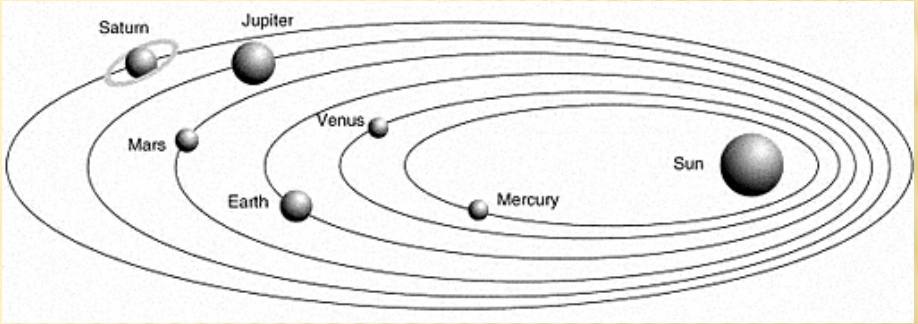
http://www.physicsdemo.com/xlab-projectile.htm

http://historytogo.utah.gov/wntrrecskijump.html

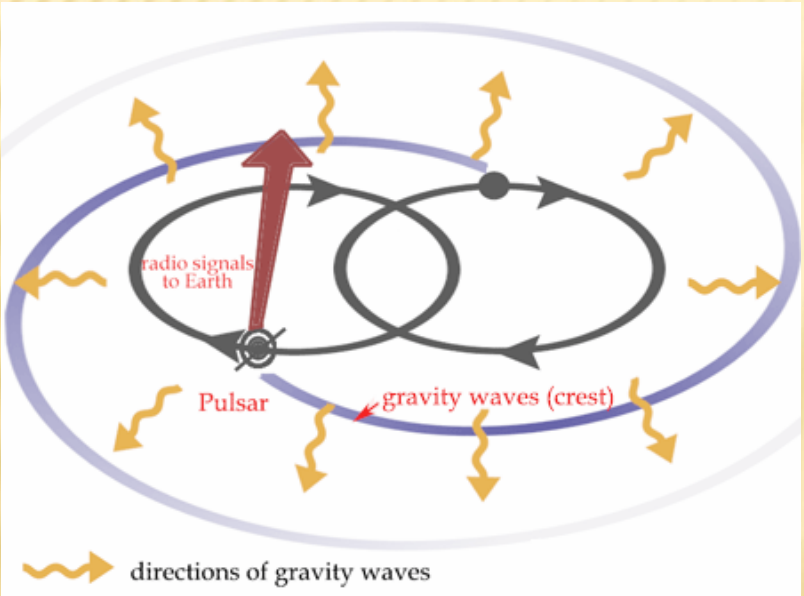
http://ffden-2.phys.uaf.edu/211_fall2004.web.dir/Brendan_Karchere/Body%20Pages%20(2-5)/Page%203.html

PHYSICAL MODELS

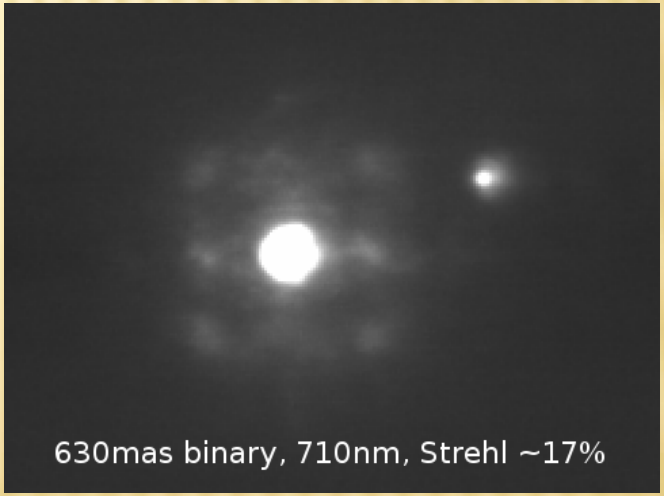
- Gravitation and Planetary Motion



http://www.bnsc.gov.uk/lzcontent.aspx?nid=4709



http://resources.edb.gov.hk/physics/articleIE/binarystar/binarystar_e.htm

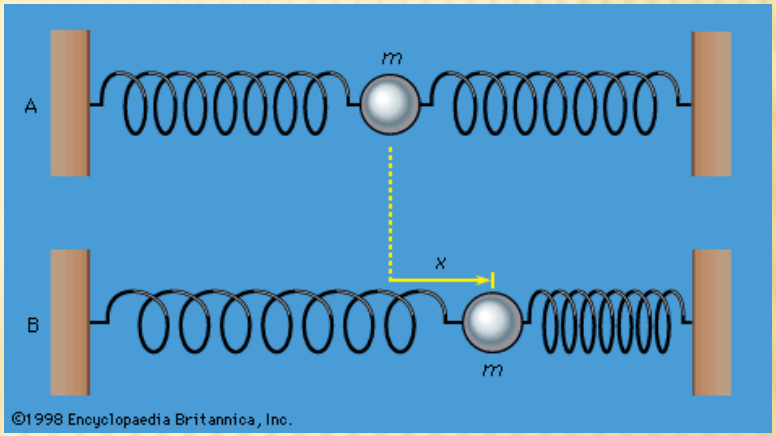


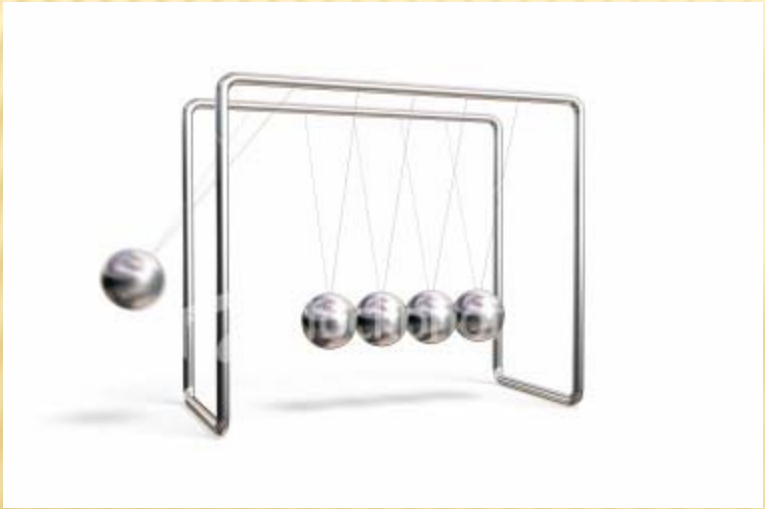
http://www.astro.caltech.edu/~nlaw/lucky_palomar/

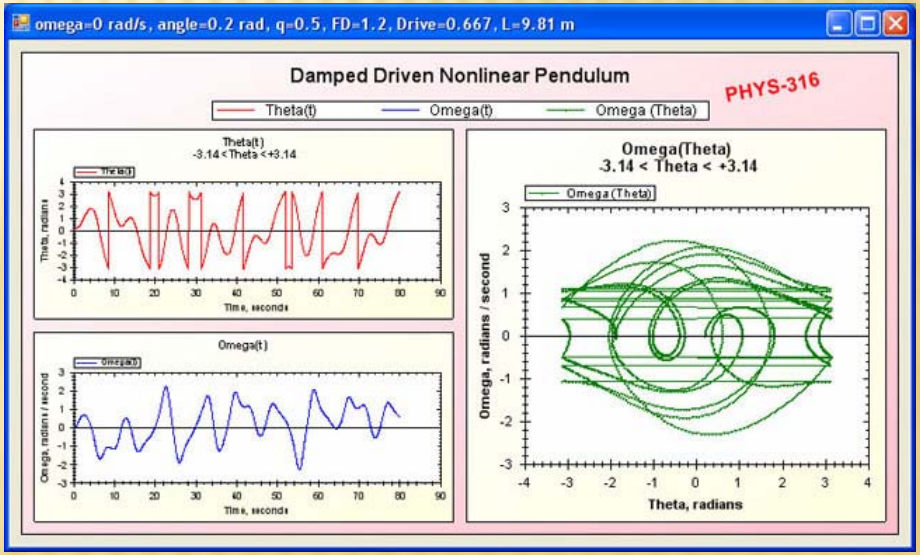
PHYSICAL MODELS

Oscillations

- Mass-Spring
- Pendula
- Nonlinear Systems





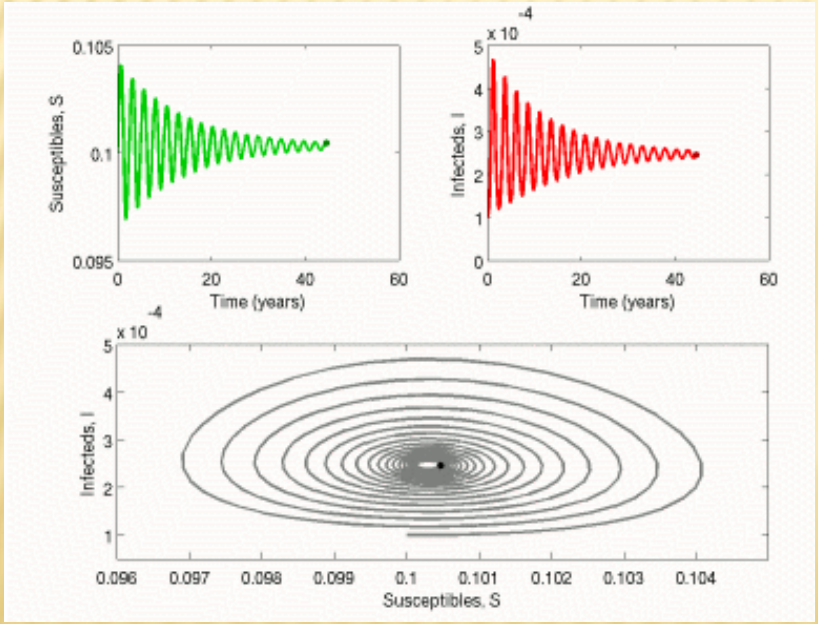


http://www.energeticforum.com/renewable-energy/1222-gravity-motor-what-do-you-think.html

http://physics.wku.edu/~barzilov/phys316/phys316aid.html

EPIDEMICS

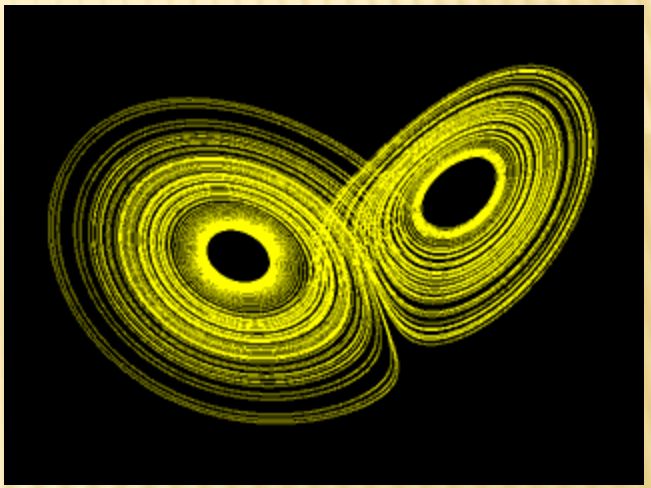
transmission dynamics of
- a communicable
- diseases

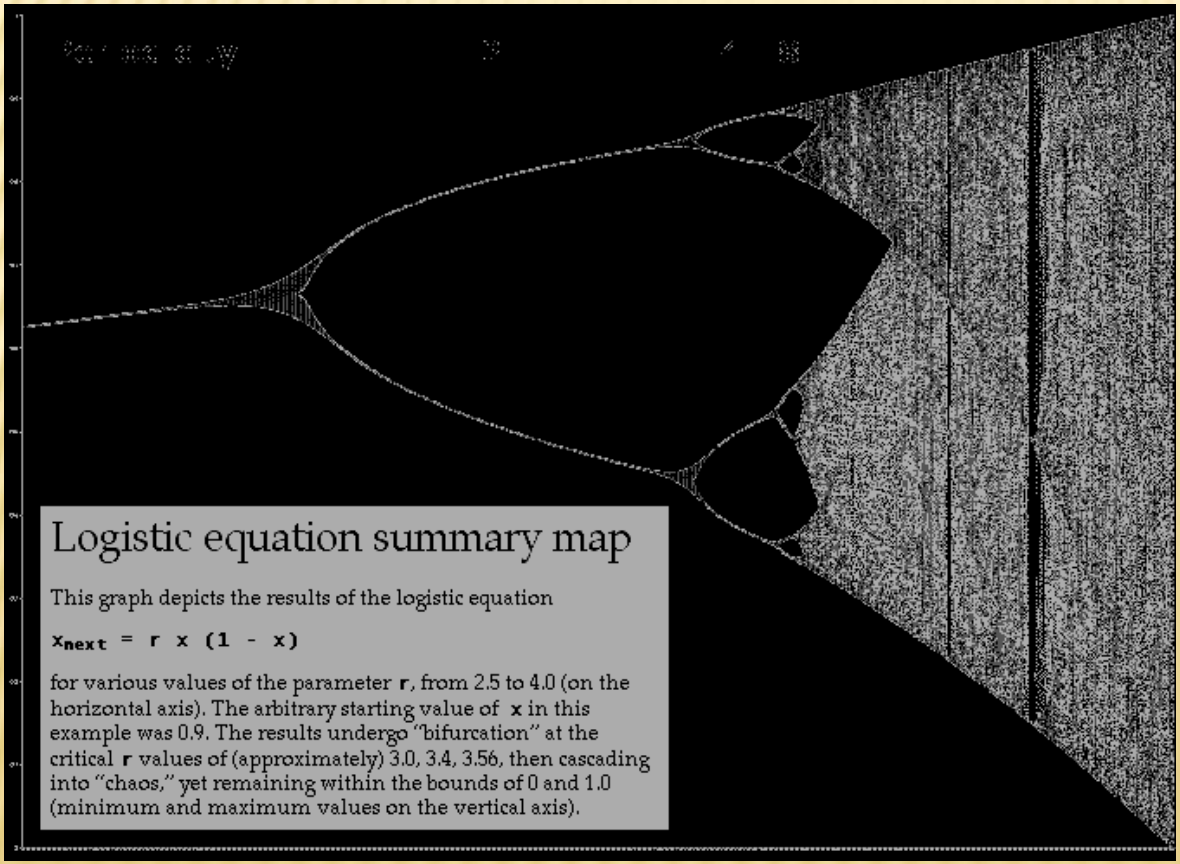


http://www.scholarpedia.org/article/Image:Animated_SIR_Epidemic.gif

CHAOS

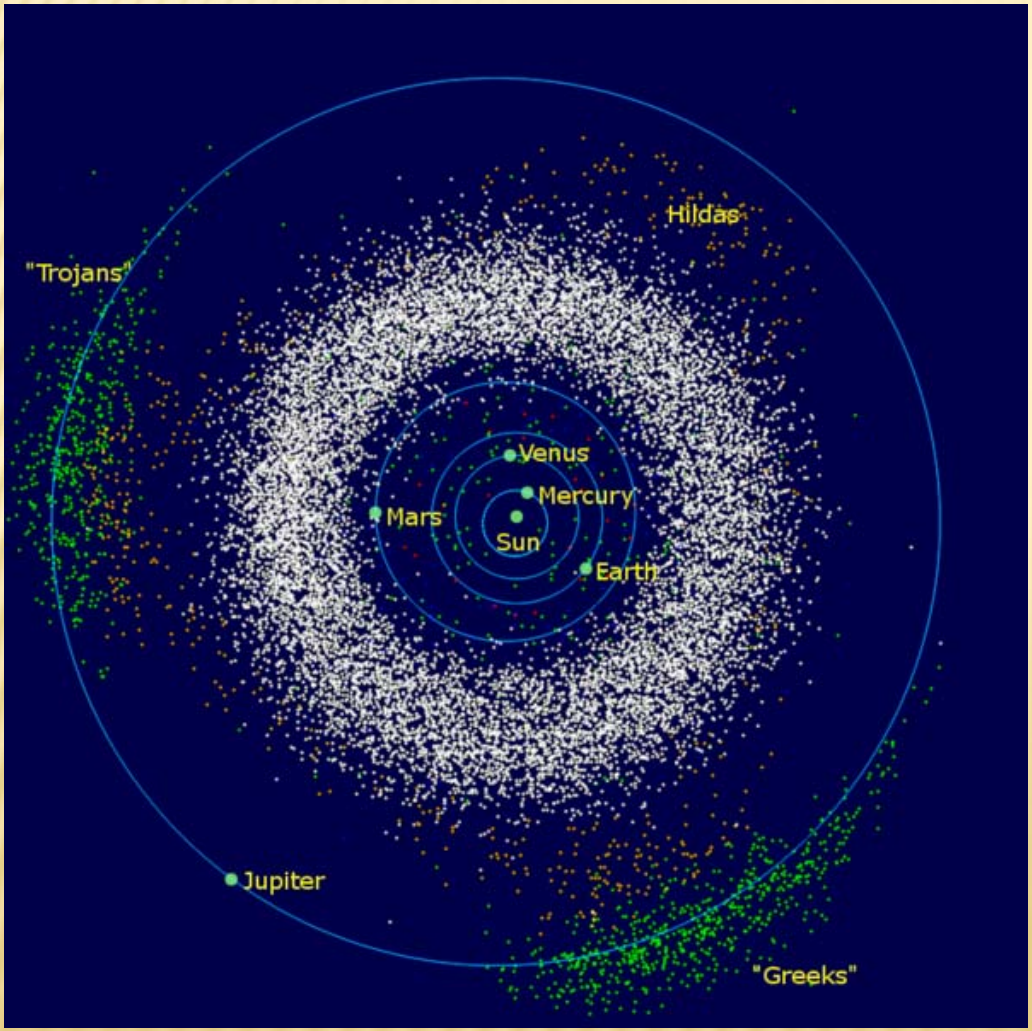
Chaotic Dynamics



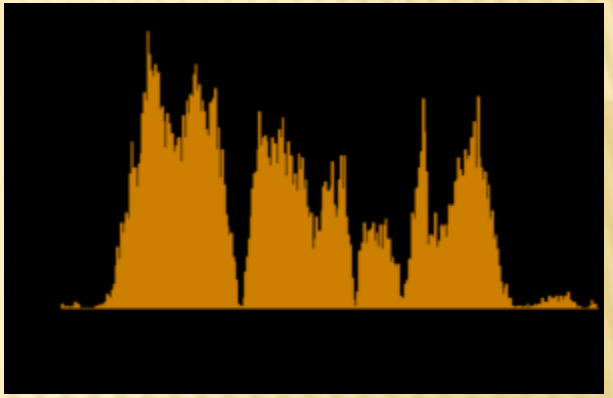


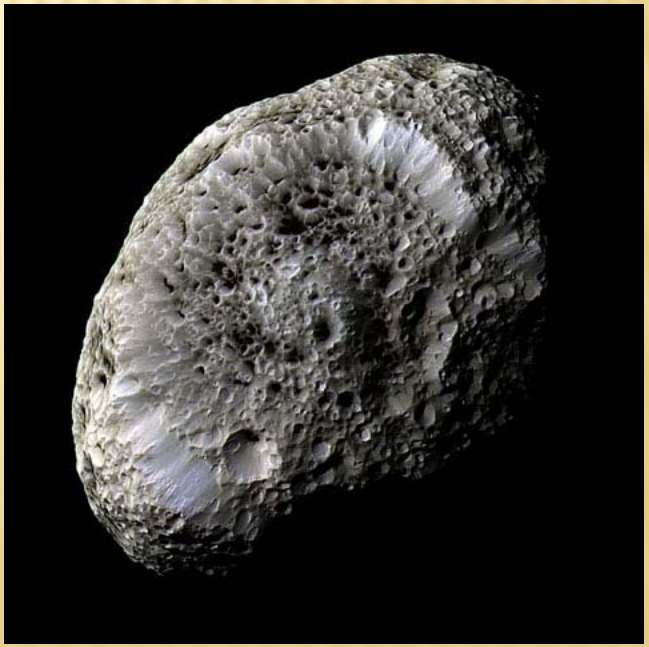
http://www.andrewclem.com/Chaos.html

CHAOS IN THE SOLAR SYSTEM



http://en.wikipedia.org/wiki/Asteroid_belt

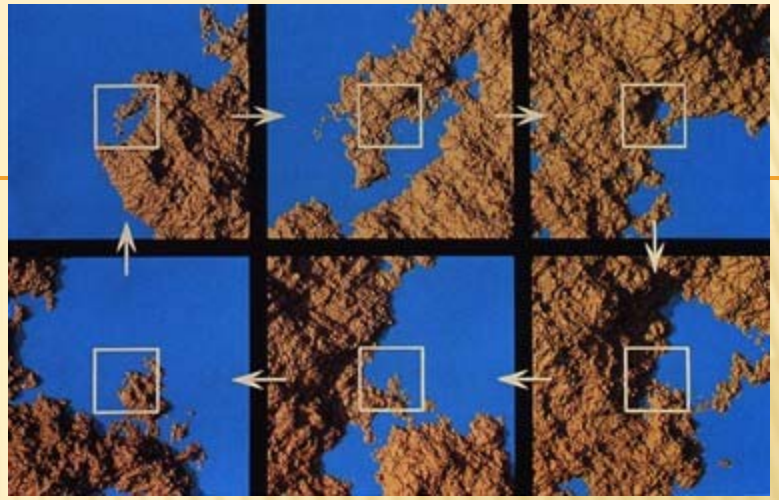




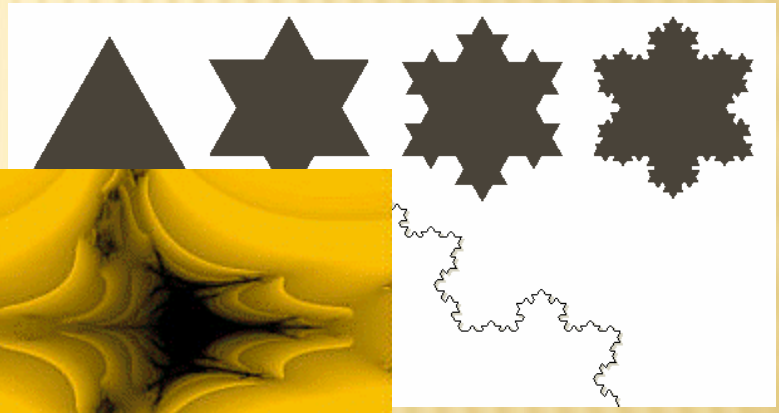
http://www.skyandtelescope.com/news/8337092.html

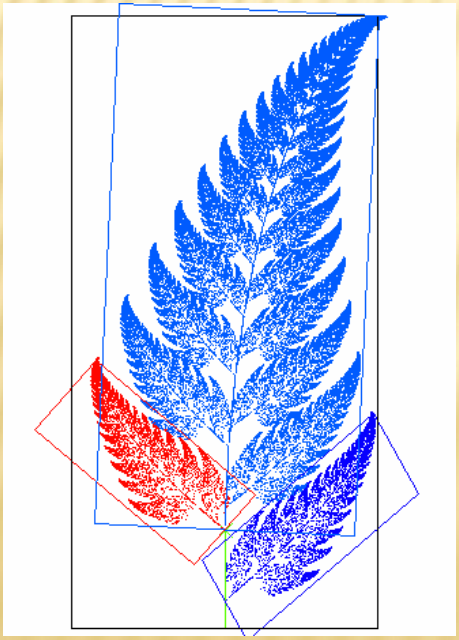
FRACTAL ANALYSIS

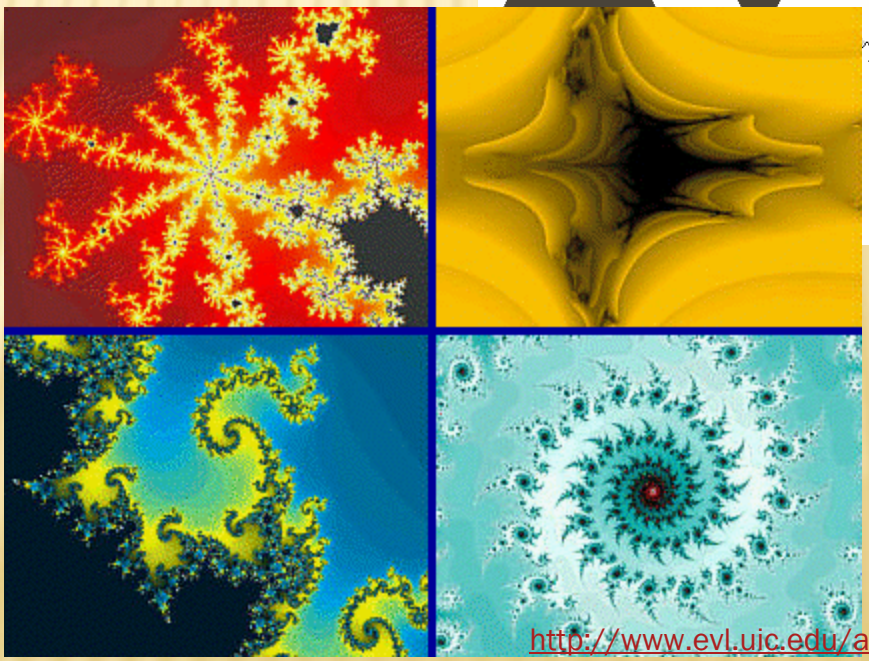
- Fractal Coastlines
- Fractal Music
- Iterated Function Systems
- Mandelbrot and Julia Sets



http://home.pacbell.net/bulens/SelfSimi.htm



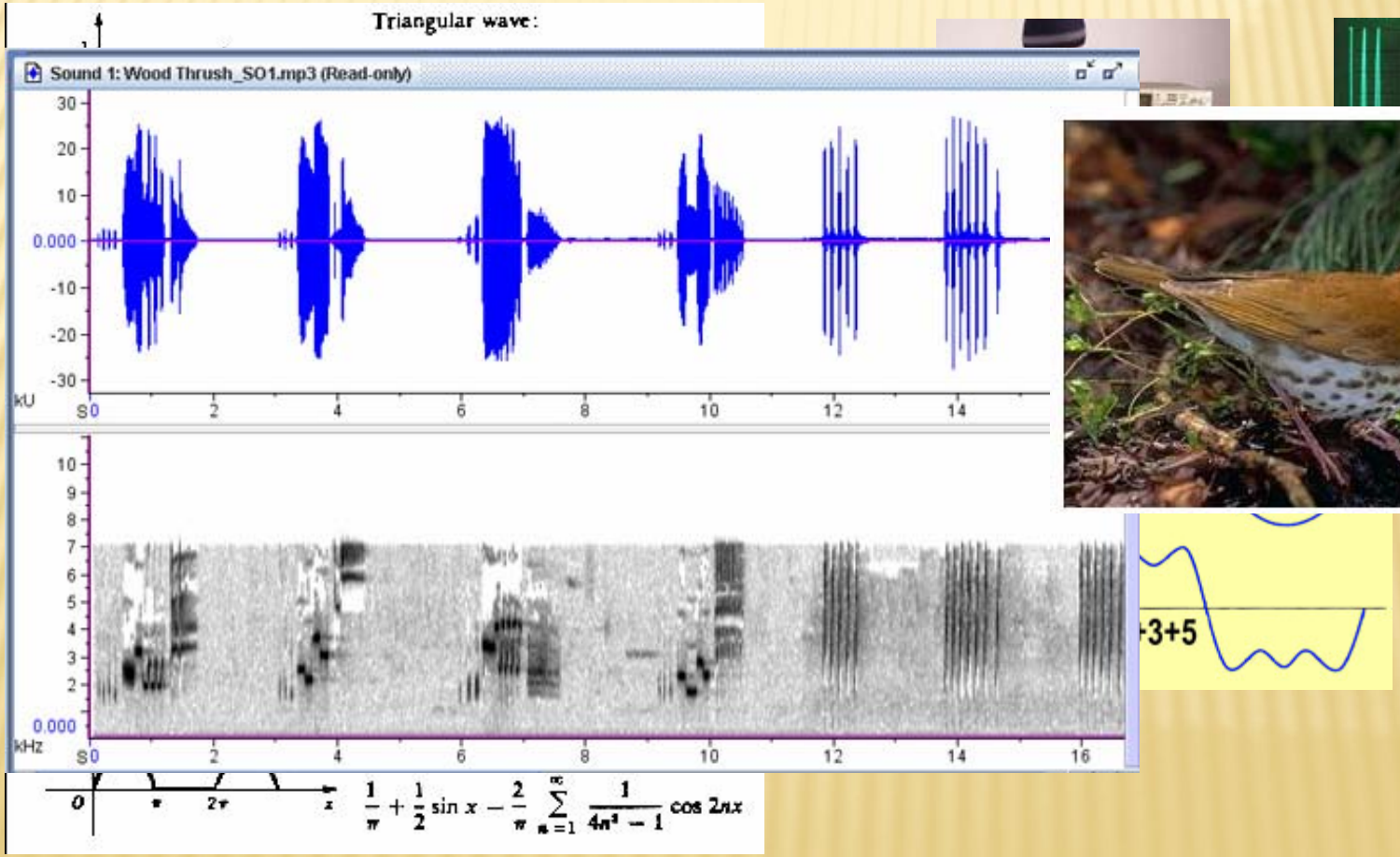




http://www.evl.uic.edu/aej/488/diagrams/fractal.gif

FOURIER ANALYSIS

Separate out harmonic frequencies from signals



http://universe-review.ca/R12-03-wave.htm

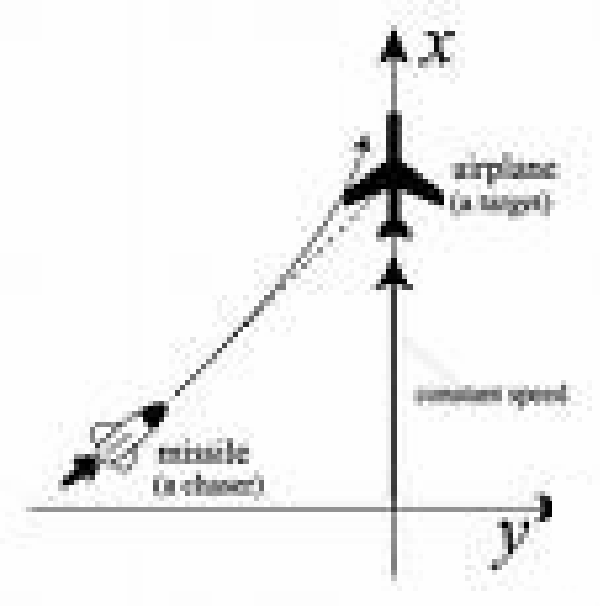
http://physics.ucsc.edu/lecturedemonstrations/waves/sound.html

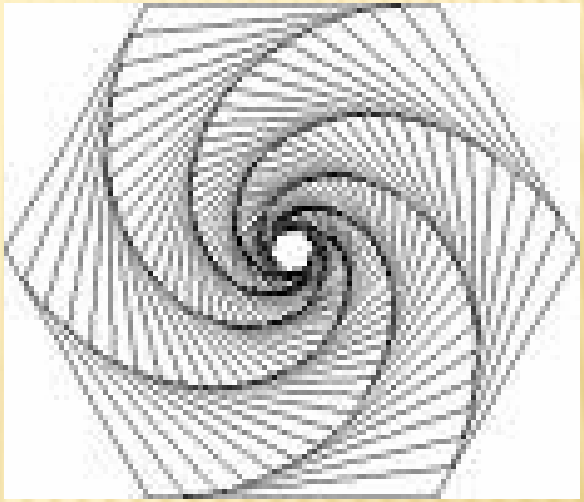
PERCOLATION MODELS

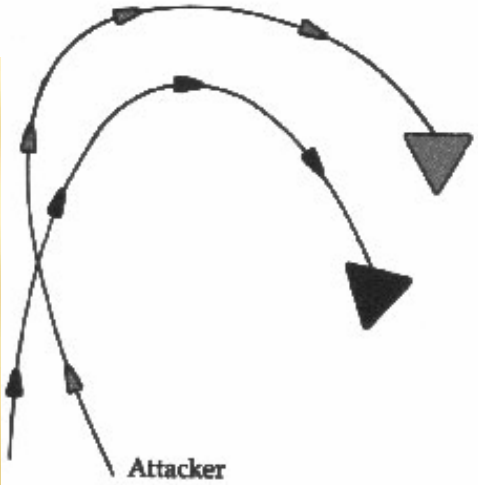
- the behavior of connected clusters in a random graph
- Examples
  - Forest Fires
  - Galaxy Formation
  - Oil Fields
  - Electrical Resistance

http://ccl.northwestern.edu/netlogo/models/Percolation

PURSUIT CURVES

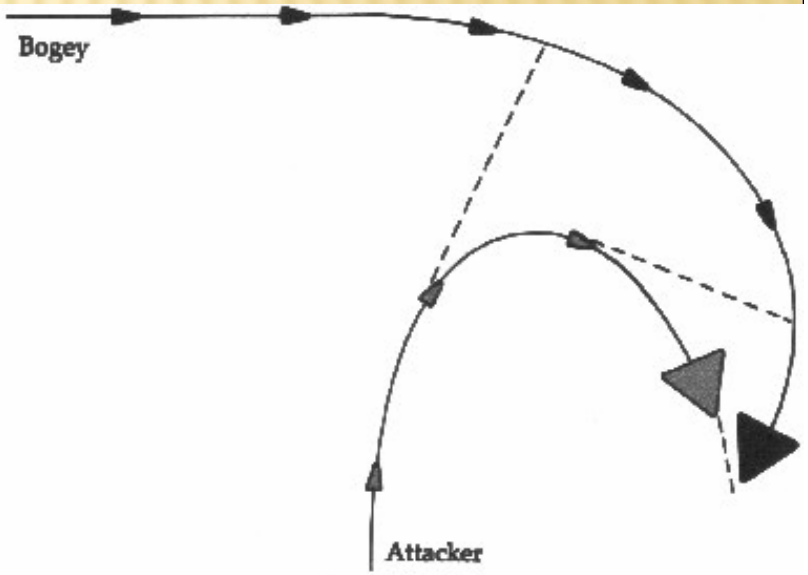






Lag-Pursuit

Nose pointed behind bogey



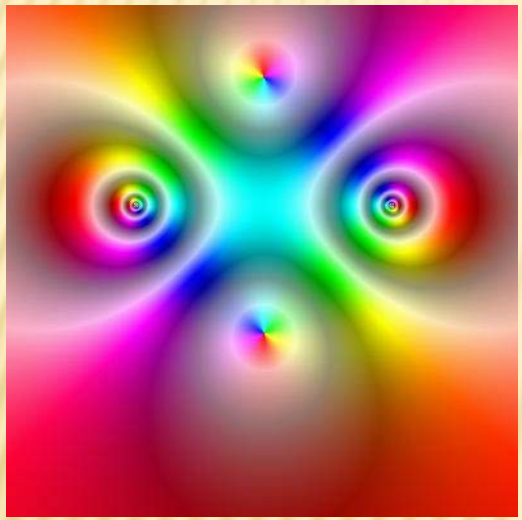
Lead-Pursuit

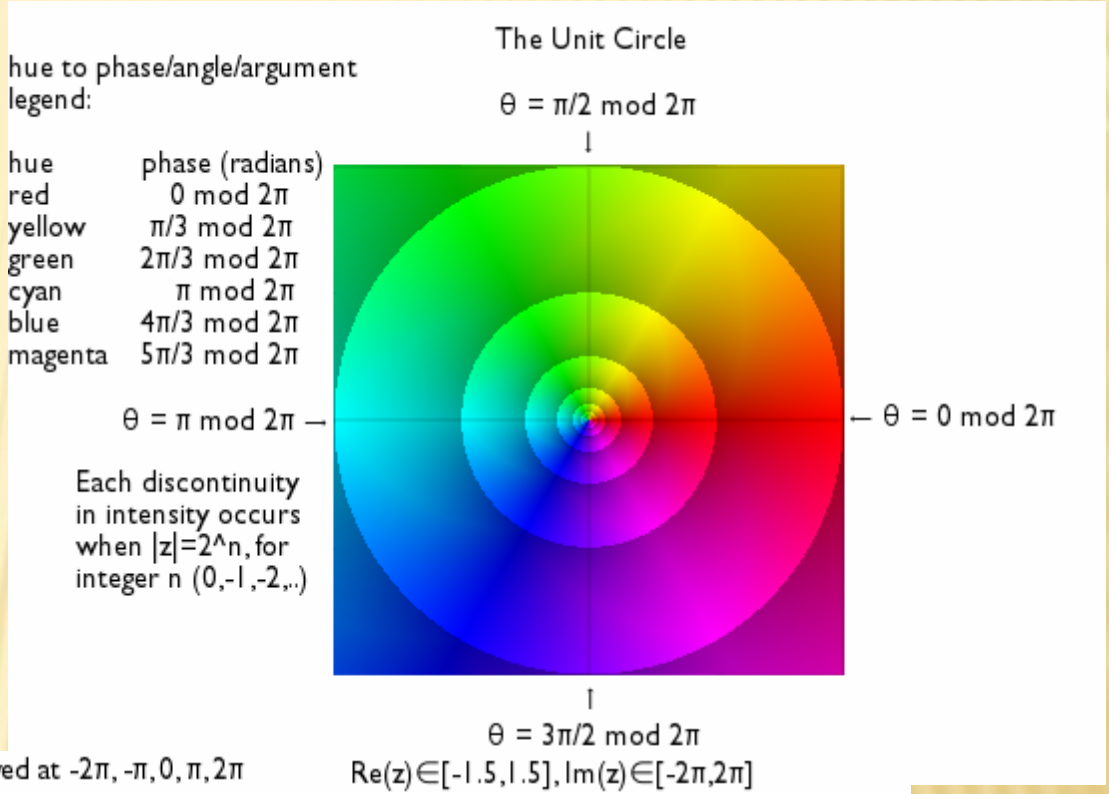
Nose pointed in front of bogey

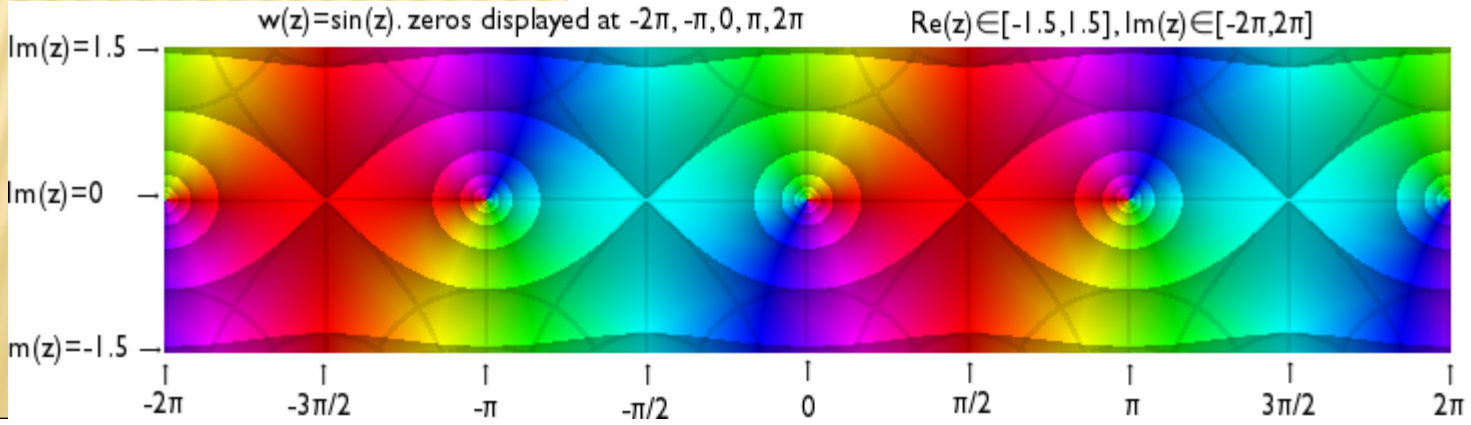
http://en.wikipedia.org/wiki/Domain_coloring

COMPLEX DOMAIN COLORING

- $z = a+ib$, $i=(-1)^{1/2}$
- $w = f(z)$

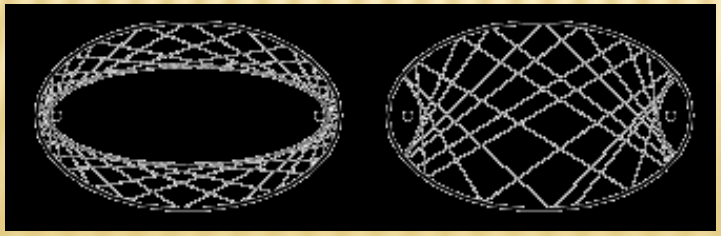






SPORTS MODELS

- Spinning Tennis Ball
- Vibrations of Hollow Core Bats
- Driving Cars with High Center of Gravity
- Projectiles with Air Drag
- Sky Diving
- And more



http://mathworld.wolfram.com/Billiards.html

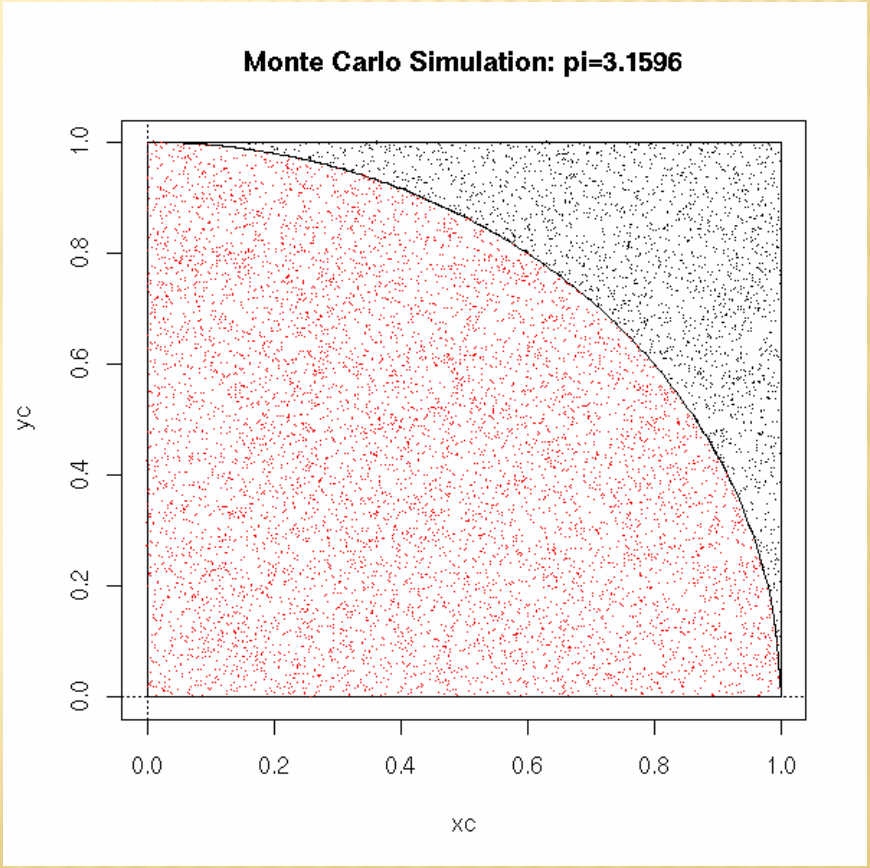
COMPUTING PI

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^n}{2n+1} + \cdots$$

- Babylonians
- Classic Formula
- Binary vs Hexadecimal
- Monte Carlo

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \cfrac{4^2}{\ddots}}}}$$

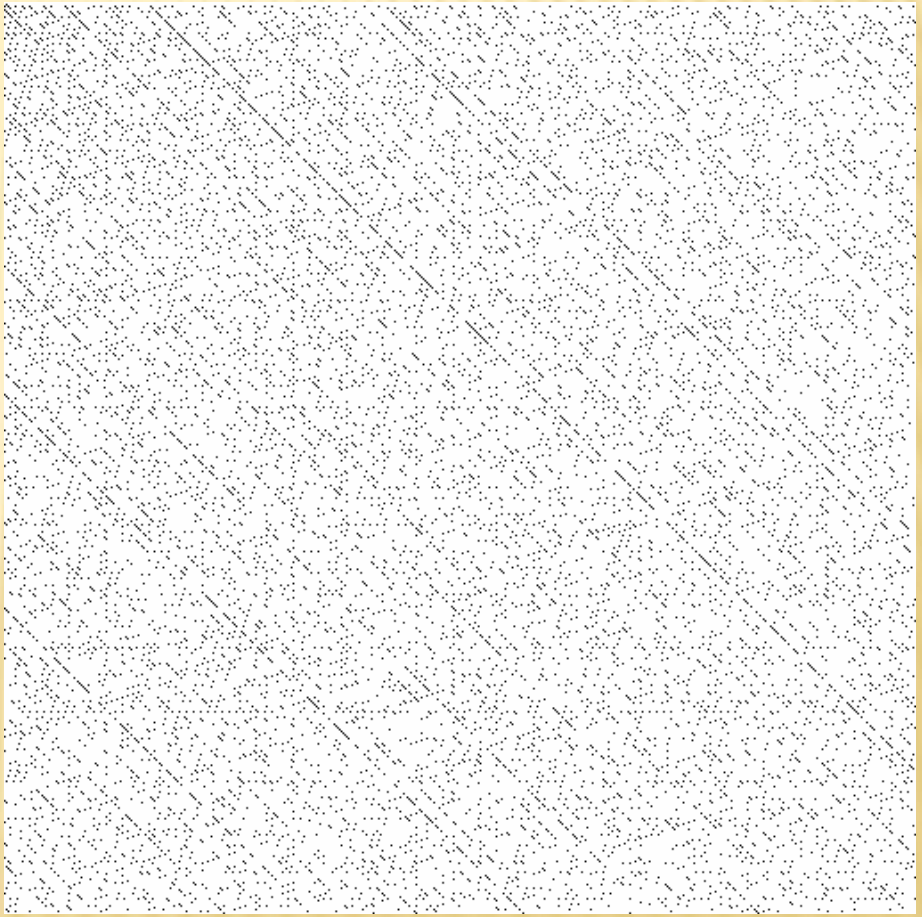
$$\pi = \sum_{n=0}^{\infty} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left(\frac{1}{16} \right)^n$$



http://zoonek2.free.fr/UNIX/48_R/16.html

DISTRIBUTION OF PRIMES

- How many primes are less than a given x?
- Are the twin primes infinite?
- Are there patterns?



TWO MATHEMATICIAN PROBLEM

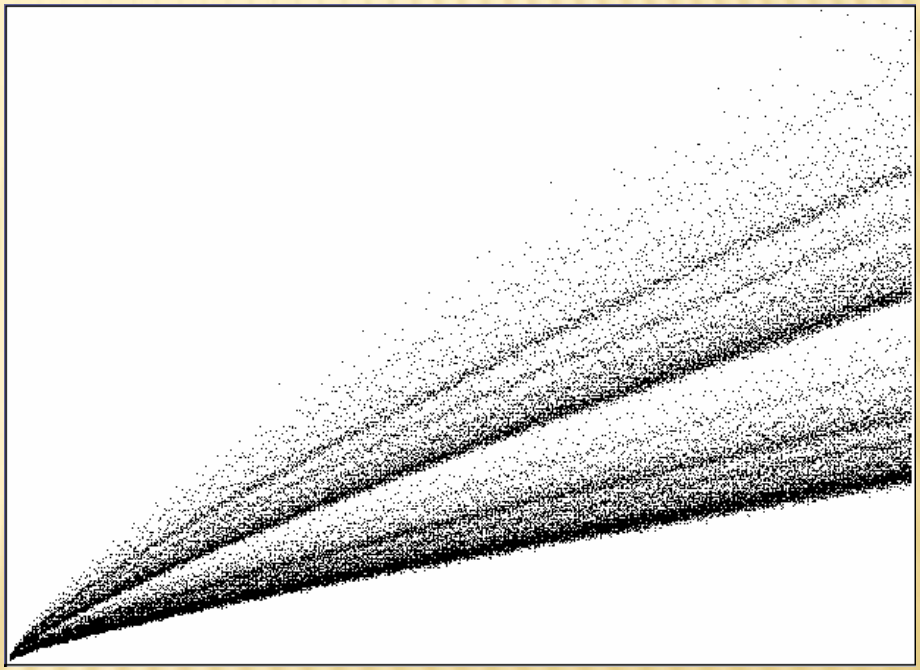
There are 2 mathematicians S and P. S knows the sum of two numbers and P knows the product of those same two numbers. Assume that the numbers are integers greater than 1 and that the mathematicians know this. For simplicity, let the first number be less than or equal to the second. Furthermore assume that each mathematician only speaks truly.

The conversation goes as follows:

S: I know that you don't know what the two numbers are.
P: Now I do know what the two numbers are.
S: Now I know what the two numbers are.

What are the two numbers?

http://homepage.mac.com/billtomlinson/primes.html



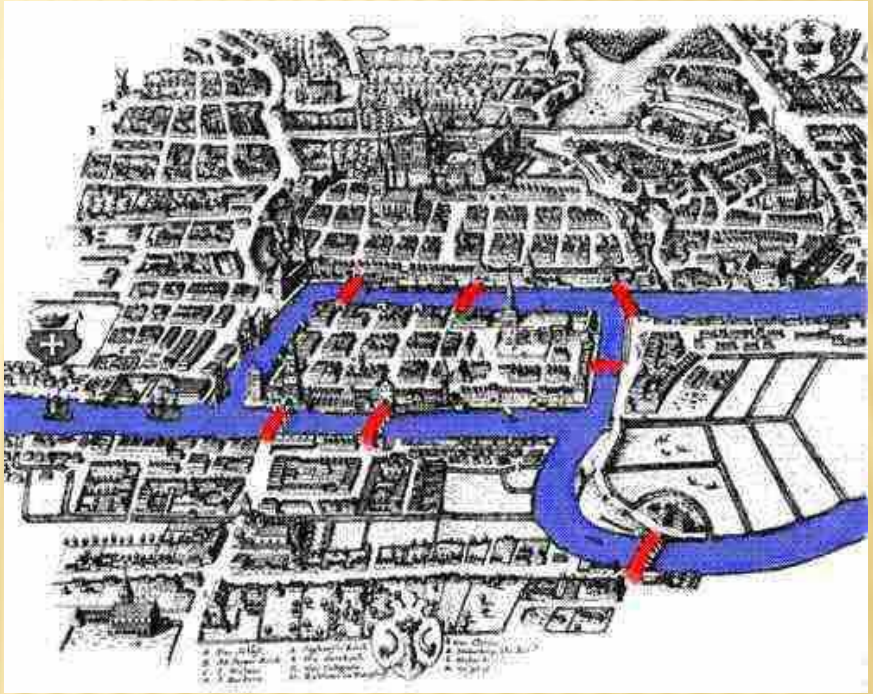
Are there even numbers unexpressible as the sum of two primes? For example, there are four ways to sum up 2 primes to 36:

5+31, 7+29, 13+23, 17+19.

THE SEVEN BRIDGES OF KÖNIGSBERG

Prelude to Graph Theory

The city of Königsberg, Prussia (now Kaliningrad, Russia) is on the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.



The problem is to decide whether it is possible to walk a route that crosses each bridge exactly once.

http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

RANDOMNESS

CELLULAR AUTOMATA

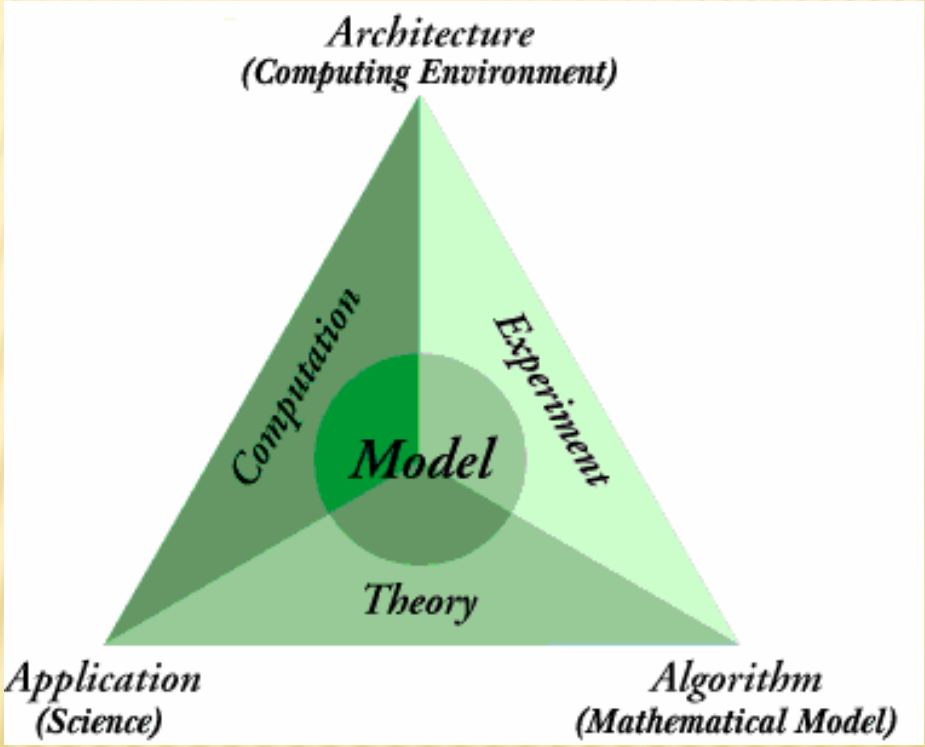
WHAT DOES ONE NEED FOR MODELING?

METHODS IN SCIENCE

- Types
  - Observation
  - Experiment
  - Computation
  - Theory
- Processing
  - Simulation
  - Visualization
  - Data Analysis
- End Products
  - Prediction
  - Verification
  - New Models

COMPUTATIONAL SCIENCE

Using Computers to do science … Shodor



SOFTWARE CLASSIFICATION

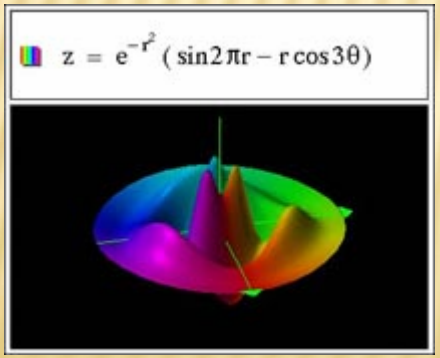
- Numerical Computation

  3*5+16 =

- Symbolic Analysis

  (x+y)^2 =

- Visualization



WHAT MATHEMATICS IS NEEDED?

- Algebra
- Trigonometry
- Graphing Functions
- Calculus
- Linear Algebra
- Differential Equations
- … and more

COURSE OUTLINE

- Week 1
  - Introduction to several models and software tools
- Week 2
  - Additional Models, form groups and pick topics, group interviews
- Week 3
  - Group research begins, start presentations, finalize abstracts
- Week 4
  - Finish project and presentation
- July 19th – Final Presentation

And now …

MODELING BY EXAMPLE

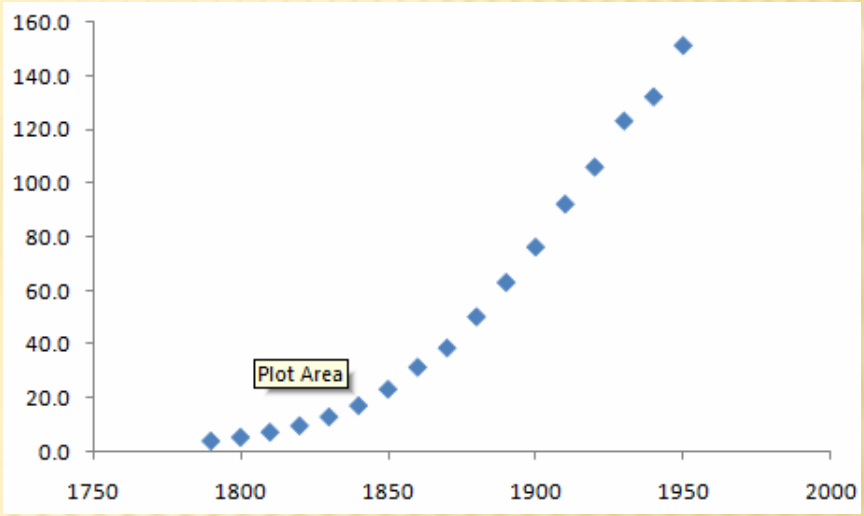
PROBLEM

Given a roll of paper find (without unrolling it) the total length of the paper on the roll. A single sheet may be provided for measuring.

POPULATION MODELS

U.S. POPULATION GROWTH

Year	Population	Year	Population
1790	3,929,214	1880	50,189,209
1800	5,308,483	1890	62,979,766
1810	7,239,881	1900	76,212,168
1820	9,638,453	1910	92,228,496
1830	12,866,020	1920	106,021,537
1840	17,069,453	1930	123,202,624
1850	23,191,876	1940	132,164,569
1860	31,443,321	1950	151,325,798
1870	38,558,371		



Rate of Change of a Quantity = Rate in – Rate out

- Rate of Change = (Population Change)/Time
- Rate in = Birth rate = bP
- Rate out = Mortality rate = dP

DERIVATION

Rate of Change of a Quantity = Rate in – Rate out

$$\frac{\Delta P}{\Delta t} = bP - dP \equiv kP$$

Let

- Time = [0, t] in N steps: $\Delta t = t/N$
- $P_n = n\Delta t$, n=0,1,2,...,N

$$\frac{P_1 - P_0}{t/N} = kP_0 \Rightarrow P_1 = P_0 + \frac{kt}{N} P_0$$

$$P_2 = \left(1 + \frac{kt}{N}\right) P_1 = \left(1 + \frac{kt}{N}\right)^2 P_0$$

$$P_N = \left(1 + \frac{kt}{N}\right)^N P_0$$

DERIVATION (CONT'D)

$$P_j = \left(1 + \frac{kt}{N}\right) P_{j-1} \Rightarrow P_N = \left(1 + \frac{kt}{N}\right)^N P_0$$

- As N gets large and Δt gets small, P_N is what?

$$P(t) = P_0 e^{kt}$$

CURVE FITTING

- Plot Data
- Fit given function – Regression Curve
- Determine Parameters (like *k*)
- Modifying Model
- Use Graphing Calculators or Software

LOGISTIC GROWTH

- Need to account for competition
- Rate out = Mortality rate = (d-mP)P =dP-mP^2

$$\frac{\Delta P}{\Delta t} = bP - dP - mP^2 \equiv kP - mP^2$$

$$P = \frac{kP_0}{(k - mP_0)e^{-kt} + mP_0}$$

http://www.uncwil.edu/courses/webcalc/LABS/newlabs/index.htm