Core Subject Taxonomy for Mathematical Sciences Education

Math NSDL Taxonomy Committee Report - April 2, 2002

Report also in PDF and MS Word

Final Taxonomy: Text Version, MS Word, PDF

Contents

Preamble

At the February 2002 MathNSDL Meeting the Mathematical Sciences Conference Group on Digital Educational Resources charged a Mathematics Taxonomy Committee to review the Core Subject Taxonomy for Mathematical Sciences, make changes and review any suggested changes from those in the mathematics community under specific guidelines. The committee consists of the following members: Kurt Cogswell, Terese Herrera, Russell Herman, Brandon Muramatsu, and Robby Robson of which the last two were consultants in getting the discussions started and the first three are the core subgroup of working members of the committee responsible for the current revision.

Discussions about a Core Subject Taxonomy for Mathematical Sciences Education had its roots in the work done by the American Mathematics Metadata Task Force http://www.mathmetadata.org/ammtf/ in 1999-2000. In March 2001 the Math NSDL group (Math NSDL is group of individuals connected with digital libraries containing significant online educational materials in mathematics) met at the MAA in Washington DC and took the Level I and Level II classification schemes developed by the American Mathematics Metadata Task Force and combined them into on classification scheme. This proposed taxonomy was then passed around to the various digital library groups for further modification. It had been looked at by representatives of the Math Forum, iLumina, the Eisenhower National Clearinghouse and MERLOT. The interested groups then met again in 2002 and this committee was formed to review the current proposal. This proposal is shown in Appendix A. The proposed taxonomy was placed at two websites for interested parties to read and comment upon: http://www.math.duke.edu/education/mathnsdl02/, http://mathforum.org/wiki/MathNSDL.

Numerous responses had come in until March 11th and this report is based upon these responses. Many groups were asked to respond to the proposed taxonomy, which is representative of the mathematics topics often encountered in mathematics K-16 education. The intent is that groups interested in classifying digital content with metadata can map their metadata to this core allowing for future integration into the National Digital Library. The major groups include representatives from College Board (AP Mathematics and

Statistics), Eisenhower National Clearinghouse, iLumina, MAA, Math Forum, MathDL, JOMA, MERLOT, and NCTM.

In this report, which is the first such response to comments and suggested changes in the taxonomy, you will find a reiteration of the guidelines by which all future suggested changes will be judged, a list of topics addressed and the responses by the committee. Also, we list excerpts from the emails to show what comments were made. This will hopefully help in eliminating future suggestions along the same lines. Finally, we present the revised taxonomy, which we deem as the core taxonomy to be disseminated amongst the mathematic community and not needing any further major revisions.

Committee Guidelines

The committee has looked at the suggestions to see if there is a need to make more modifications. Modifications are typically made only if it is not possible to "live with the taxonomy" as the taxonomy stands. There have to be good reasons to make changes. This means that people cannot "map" their data to the core taxonomy. There are many ways to do this and the purpose of this taxonomy is to begin to identify a common set of terms. Individual libraries can set up their own structures, which can be browsed using their own tools. Thus, it is expected that there will be plenty of flexibility in the ordering of topics within the core taxonomy and that various groups may vary considerably in their own ordering. What matters is if the topics exist somewhere in the listing. However, the committee has taken seriously all of the suggestions and made changes when they were simple modifications and not prone to further comments.

The committee had decided to resist suggestions that included topics that properly belonged elsewhere in the metadata, e.g., level, intended audience, type of material, etc. Also, the order in which topics are listed should make little difference if the real purpose is to match search keys. It might be more relevant for a browse. People who have experience in working libraries often find that users are more likely to find what they want by search rather than browse. However, the committee also needed to be fair to those offering suggestions and was committed to presenting a taxonomy that would need no major revisions after this time.

As indicated in Appendix A, the February 2002 proposed taxonomy, there are some goals that we have kept in mind in our review of the comments and suggestions:

Primary Goal:

To create a subject taxonomy for the mathematical sciences to which each participating digital library/collection is able to map their internal subject taxonomy.

Secondary Goals:

- No more than approximately 9 topics per level (for human browsability).
- End-user may use this taxonomy to catalog their resource.
- This taxonomy may be implemented in a browse structure.
- A digital library/collection may implement this as their vocabulary.
- Arbitrary levels of detail are allowed below the tree structure indicated.
- Changes to this taxonomy can be proposed and will follow a given easy revision process.
- Topic names should include no punctuation and these names should be kept as short as possible.

Suggested Topics

Several topics came up in the solicited emails. We have grouped them into major topics and the committee responses are provided below. In some cases the topics are simple and specific. Other topics were broad and more involved. The following list hopefully conveys the topics that various groups have made suggestions about. The final list of proposed changes can be found in Appendix E and the new taxonomy is listed in Appendix F.

1. Pi and e
2. Repetition of Addition, Subtraction, Multiplication and Division
3. Polygons vs Quadrilaterals
4. Number of Levels, Variety of Depth – Topic vs Level
5. Numbering Problems
6. Location of Geometrical Topics
7. Location of Famous Problems
8. Curvilinear Coordinates
9. Fractal Geometry
10. Fourier Series and Fourier Transforms
11. Brownian Motion
12. Repetition of Topics (like Patterns and Sequences)
13. Roots, Exponents and Powers
14. Probability and Statistics
    a. Central Tendency
    b. combinations, permutations, choosing
15. Elementary Mathematics vs Number and Computation
16. Discrete Mathematics
17. Geometry and Topology
18. Measurement
19. Algebra and Number Theory
20. Functions and Equations
21. Graphing
22. Systems of Equations
23. Estimation

Committee Responses

In this section we provided a list of responses made to the suggestions that we had received. We have included excerpts of the comments made by individuals in the next section. For a complete list of changes and the final taxonomy, see the last two appendices. Some changes have been made there that do not show up in the list. Those were a result of discussions between the three main Committee members.

Simple/No Changes

1. Pi and e
   Suggestion: Place under a new heading of Famous Problems and add other famous numbers.
   Justification: These are specific numbers and not at the same level in this category as other items. Also, there was some concern about putting them under irrational or transcendental. The committee felt this classification made the most sense and provided an place for other similar numbers.
2. Polygons vs Quadrilaterals
   Suggestion: Leave Alone
   Justification: The current listings under geometry were typical of several geometry texts in which Polygons was a major group. One could take all quadrilaterals out of this group and call it Other Polygons with a Quadrilateral Group at the level of Triangles, which is not done in texts, or put them as a subgroup of Polygons. However, this would force a 5^{th} level. The Committee chose to eliminate the detail and just leave the classification as Polygon/Regular allowing individual libraries to easily map into this section.
3. Location of Famous Problems
   Suggestion: Delete Famous Problems from Number Theory
   Justification: Typically Famous Problems are historical and span many areas of mathematics. They belong in the Mathematics History Section.

4. Curvilinear Coordinates
   Suggestion: Move to Advanced Calculus after Stoke's Theorem.
   Justification: This is one subject under which curvilinear coordinates is covered in detail, though technically they are seen much earlier in the form of polar, cylindrical and spherical coordinate systems.
5. Fractal Geometry
   Suggestion: Leave alone
   Justification: This is an instance of how one can locate simple fractal concepts without sophistication. There is one other location in Dynamical Systems. However, there are fractal structure that are not considered as dynamical systems, but as geometry.
6. Brownian Motion
   Suggestion: Leave alone
   Justification: This is a major topic in Stochastic Processes and is just as much physics as calculus or differential equations can be.
7. Discrete Mathematics
   Suggestion: Leave Alone.
   Justification: The topics chosen are those of a text in Discrete Mathematics, which is how many of the sublists have been created. This topic was originally listed under a section called Algebra and Discrete Mathematics. It's movement left it looking sparse. The response was not specific enough to add more subtopics.
8. Algebra and Number Theory
   Suggestion: Leave Alone
   Justification: This is a simple case in which mapping into the taxonomy is possible without any changes. Number Theory has few subtopics and splitting these could lead to similar splittings of many other topics, leading to too many topics at this level.
9. Functions and Equations
   Suggestion: Leave alone
   Justification: While the combination of these topics into one would simplify the structure, there really is no need to change the structure. Functions and equations are related, but different topics. A quadratic function can be graphed and studied but a quadratic equation need not be related to a graph in looking at solution techniques.
10. Graphing
    Suggestion: Change to Graphing Techniques.
    Justification: This topic on graphing is not about graphing specific lines, etc, but about graphing in general, such as graphing techniques and instructions. Perhaps a change to Graphing Techniques is in order. That is why is looks bare. More specifically, this section was meant to be for general graphing techniques with graphing tools or for topics like symmetries (even/odd functions), translations, rotations, reflections, asymptotes, etc. This also lead to the observed need for a section on Transformations in Plane Geometry.
11. Systems of Equations
    Suggestion: Leave alone
    Justification: Systems of equations is already in algebra/equations/systems and algebra/linear algebra/systems of linear equations, in which one case is more about solution and the other about structure.

More Complicated Changes

1. Elementary Mathematics vs Numbers and Computation
   Suggestion: Change the Elementary Mathematics section to Numbers and Computation and move geometric topics to the Geometry section.
   Justification: The Committee agreed that this was more natural and helped to convey the idea that this section was not intended to indicate the level of instruction. The committee adapted some of the suggestions made by the NCTM (See Appendix C) and left some of their suggestions to the mapping of the metadata. Stripping away levels has eliminated the need for some of the redundancies that were commented upon.

2. Roots, Exponents and Powers
   Suggestion: Replace Square Roots with Roots and under Exponents add subtopics with Integer, Fractional and Negative as per NCTM.
   Justification: This would eliminate the confusion of simple roots with other exponent manipulations.
3. Measurement
   Suggestion: Add missing elements: Money, Temperature, Weight and Mass, Speed, Areas of Triangles, Areas of Rectangles, Polygons changed to Other Polygons, and move Scale.
   Justification: The mapping seems clear except for a couple of topics. Some are definitely missing. Furthermore, there is a strong online presence of several of these topics. We did not move the entire topic of Measurement as suggested by NCTM. Its subtopics could fit under Numbers and Computation as a mathematical application and we wanted to limit the number of top level headings.
4. Estimation
   Suggestion: Move Estimation under Operations.
   Justification: We saw no need to add Estimation subtopics to each category as suggested by NCTM or even the subtopics under estimation. Though estimation is not a typical Operation, it can be argued that it is a means of computation and may not belong at a higher level than this.
5. Geometry and Topology
   Suggestion: Change Geometry to Geometry and Topology
   Justification: This is a simple change. However, there is still a problem with Trigonometry in the same section. We have opted to leave Trigonometry alone.
6. Location of Geometrical Topics
   Suggestion: Move geometric topics to Geometry and add Number Patterns.
   Justification: Geometric topics do not belong under Numbers and Computation.
7. Fourier Series and Fourier Transforms
   Suggestion: Fix numbering and split Signal Analysis from Transforms.
   Justification: The original topic list had Signal Analysis with subtopics Fourier Series thru Image Processing. This should fix most concerns. Also, Fourier Series appears as a topic in Advanced Calculus as well and would naturally appear in many other places, like under pde's as a lower level than those listed. There is a field of Transform Analysis which can be viewed differently than just signal processing. Splitting the topics would be more natural.
8. Probability and Statistics
   Suggestion: Trim down the graphing subtopics, reduce the number of levels by moving mean, median and mode and move things around, like expectation value. Generally move a few topics and introduce Elementary Topics in Probability and Statistics.
   Justification: There needed to be some reworking of these topics after the Data Analysis topics were moved here at the DC meeting. Also, we had to keep in mind where some items might naturally be found by browsers, such as combinations and permutations, which also appear in Discrete Mathematics. Though there is some recognition that some topics in the Elementary groups might be handled using other metadata, this would help with the mapping of common topics which might lie at this higher level in the K-12 curriculum.

General Comments

1. Repetition of Addition, Subtraction, Multiplication and Division
   Comment: Leave topics under each level, but arrange common operations to always be at the top of the list. This will allow for specialized uses of the particular operations under the tree structure.
2. Number of Levels, Variety of Depth – Topic vs Level
   Comment: Our goal is not to create a completely logical outline of mathematics, but rather a Search List appropriate for individual topic searches as well as menu searches. Number of levels [headings] at 0 and 1 levels followed our thoughts on browsability [10 to 11 max]. Some areas have more levels under them than others; those with few sub-levels can be enlarged by individual libraries as appropriate for their users. For the most part, we expect more resources in some areas. It does not make sense to provide a lot of depth in areas that will not have a large number of

resources. Should some areas later need more subtopics, then there will be procedures in place for the committee to review requests to add the depth.

3. Repetition of Topics (like Patterns and Sequences)
   Comment: There are times when repetition is good. We would like to keep redundancies to a minimum, except where it is most useful or there are fundamentally different uses of the terminology in different fields, such as the different use of systems of equations or of fractals, for example.

Appendices

A. "Core" Subject Taxonomy for Mathematical Sciences Education

Proposed by the Mathematical Sciences Conference Group on Digital Educational Resources
February 2002

Brief Background:
A number of collections and organizations with a strong interest in using technology to enhance mathematical science education have worked to develop the "core" subject taxonomy for mathematical science education presented here.

Primary Goal:

To create a subject taxonomy for the mathematical sciences to which each participating digital library/collection is able to map their internal subject taxonomy.

Secondary Goals:

No more than approximately 9 topics per level (for human browsability)
End-user may use this taxonomy to catalog their resource
This taxonomy may be implemented in a browse structure
A digital library/collection may implement this as their vocabulary
Arbitrary levels of detail are allowed below the tree structure indicated
Changes to this taxonomy can be proposed and will follow a given easy revision process

Ground Rules for editing this document: (*Added for use at Wiki site.*)

Add two asterisks, your email address and the date after a change in the hierarchy.

Record a description of your changes in the "Record of Changes" section, including your email and date of making the change.

(Note: This document was created in Microsoft Word and saved as a HTML/Web document.)

Record of Changes:

Moved ElementaryMathematics.Irrational.Algebraic, ElementaryMathematics.Irrational.pi and ElementaryMathematics.Irrational.e up one level in the taxonomy. (Working Group, 2/15/02)

Made new category Discrete Mathematics, put after Algebra and Number Theory (Working Group, 2/15/02)

Renamed and Algebra and Discrete Mathematics to Algebra and Number Theory (Working Group, 2/15/02)

Added DiscreteMathematics.Chaos (Working Group, 2/15/02)

Moved ElementaryMathematics.Data to StatisticsAndProbability.Data (Working Group, 2/15/02)

Added Geometry.FractalGeometry (Working Group, 2/15/02)

Change DifferentialEquations to DifferentialAndDifferenceEquations (Working Group, 2/15/02)

Changed DifferentialEquation.DynamicSystems.DifferentialDynamics to DifferentialEquation.DynamicSystems..DifferentiableDynamics (Working Group, 2/15/02)

Removed Analysis.NumericalAnalysis.Miscellaneous (Working Group, 2/15/02)

Added MathematicsHistory.General (Working Group, 2/15/02)

B. Proposed Taxonomy in DC

- 1.0 Elementary Mathematics
  - 1.1 Numbers
    - 1.1.1 Natural
    - 1.1.2 Integers
    - 1.1.3 Rational
    - 1.1.4 Irrational
    - 1.1.5 Algebraic
    - 1.1.6 pi
    - 1.1.7 e
    - 1.1.8 Real
    - 1.1.9 Complex
  - 1.2 Arithmetic
    - 1.2.1 Operations
      - 1.2.1.1 Addition
      - 1.2.1.2 Subtraction
      - 1.2.1.3 Multiplication
      - 1.2.1.4 Division
      - 1.2.1.5 Square Roots
      - 1.2.1.6 Factorials
    - 1.2.2 Fractions
      - 1.2.2.1 Equivalent Fractions
      - 1.2.2.2 Addition
      - 1.2.2.3 Subtraction
      - 1.2.2.4 Multiplication
      - 1.2.2.5 Division
      - 1.2.2.6 Ratio and Proportion
    - 1.2.3 Decimals
      - 1.2.3.1 Addition
      - 1.2.3.2 Subtraction
      - 1.2.3.3 Multiplication
      - 1.2.3.4 Division
      - 1.2.3.5 Percents
    - 1.2.4 Estimation
    - 1.2.5 Comparison of numbers
    - 1.2.6 Exponents
      - 1.2.6.1 Multiplication
      - 1.2.6.2 Division
      - 1.2.6.3 Powers
  - 1.3 Patterns and Sequences
    - 1.3.1 Geometric Patterns
    - 1.3.2 Tilings and Tessellations
    - 1.3.3 Golden Ratio
    - 1.3.4 Fibonacci Sequence
    - 1.3.5 Arithmetic Sequence
    - 1.3.6 Geometric Sequence
  - 1.4 Shapes and Figures
    - 1.4.1 Plane shapes
    - 1.4.2 Spatial Sense
    - 1.4.3 Symmetry
    - 1.4.4 Similar Figures
    - 1.4.5 Solid Shapes
  - 1.5 Measurement
    - 1.5.1 Units of Measurement
      - 1.5.1.1 Metric System
      - 1.5.1.2 Standard Units
    - 1.5.2 Linear Measure
      - 1.5.2.1 Distance
      - 1.5.2.2 Circumference
      - 1.5.2.3 Perimeter
      - 1.5.2.4 Scale
    - 1.5.3 Area
      - 1.5.3.1 Area of Polygons
      - 1.5.3.2 Area of Circles
      - 1.5.3.3 Surface Area
    - 1.5.4 Volume
- 2.0 Logic and Foundations
  - 2.1 Logic
    - 2.1.1 Venn Diagrams
    - 2.1.2 Propositional and Predicate Logic
    - 2.1.3 Induction
    - 2.1.4 Methods of Proof
  - 2.2 Set Theory
    - 2.2.1 Sets and Set Operations
    - 2.2.2 Relations and Functions
    - 2.2.3 Cardinality
    - 2.2.4 Axiom of Choice
  - 2.3 Computability, Decidability and Recursion
  - 2.4 Model Theory
- 3.0 Algebra and Number Theory
  - 3.1 Algebra
    - 3.1.1 Graphing
    - 3.1.2 Functions
      - 3.1.2.1 Linear
      - 3.1.2.2 Quadratic
      - 3.1.2.3 Polynomial
      - 3.1.2.4 Rational
      - 3.1.2.5 Exponential
      - 3.1.2.6 Logarithmic
      - 3.1.2.7 Piece-wise
      - 3.1.2.8 Step
    - 3.1.3 Equations
      - 3.1.3.1 Linear
      - 3.1.3.2 Quadratic
      - 3.1.3.3 Polynomial
      - 3.1.3.4 Rational
      - 3.1.3.5 Exponential
      - 3.1.3.6 Logarithmic
      - 3.1.3.7 Systems
    - 3.1.4 Inequalities
    - 3.1.5 Matrices
    - 3.1.6 Sequences and Series

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- 3.1.7 Algebraic Proof
- 3.2 Linear Algebra
  - 3.2.1 Systems of Linear Equations
  - 3.2.2 Matrix algebra
  - 3.2.3 Vectors in R3
  - 3.2.4 Vector Spaces
  - 3.2.5 Linear Transformations
  - 3.2.6 Eigenvalues and Eigenvectors
  - 3.2.7 Inner Product Spaces
- 3.3 Abstract Algebra
  - 3.3.1 Groups
  - 3.3.2 Rings and Ideals
  - 3.3.3 Fields
  - 3.3.4 Galois Theory
  - 3.3.5 Multilinear Algebra
- 3.4 Number Theory
  - 3.4.1 Integers
  - 3.4.2 Primes
    - 3.4.2.1 Divisibility
    - 3.4.2.2 Factorization
    - 3.4.2.3 Distributions of Primes
  - 3.4.3 Congruences
  - 3.4.4 Diophantine Equations
  - 3.4.5 Irrational Numbers
  - 3.4.6 Famous Problems
  - 3.4.7 Coding Theory
  - 3.4.8 Cryptography
- 3.5 Category Theory
- 3.6 K-Theory
- 3.7 Homological Algebra
- 3.8 Modular Arithmetic

4.0 Discrete Mathematics

- 4.1 Cellular Automata
- 4.2 Combinatorics
- 4.3 Game Theory
- 4.4 Algorithms
- 4.5 Graph Theory
- 4.6 Linear Programming
- 4.7 Order and Lattices
- 4.8 Theory of Computation
- 4.9 Chaos

5.0 Geometry

- 5.1 Plane Geometry
  - 5.1.1 Measurement
  - 5.1.2 Geometric Proof
  - 5.1.3 Parallel and Perpendicular Lines
  - 5.1.4 Angles
  - 5.1.5 Triangles
    - 5.1.5.1 Pythagorean Theorem
    - 5.1.5.2 Properties of Right Triangles
  - 5.1.6 Congruence
  - 5.1.7 Similarity
  - 5.1.8 Polygons
    - 5.1.8.1 Rectangles
    - 5.1.8.2 Squares
    - 5.1.8.3 Trapezoids
    - 5.1.8.4 Pentagons
    - 5.1.8.5 Hexagons
    - 5.1.8.6 Regular Polygons
  - 5.1.9 Circles
- 5.2 Solid Geometry
  - 5.2.1 Lines and Planes
  - 5.2.2 Angles
  - 5.2.3 Spheres
  - 5.2.4 Cones
  - 5.2.5 Cylinders
  - 5.2.6 Pyramids
  - 5.2.7 Prisms
  - 5.2.8 Polyhedra
- 5.3 Analytic Geometry
  - 5.3.1 Cartesian Coordinates
  - 5.3.2 Lines
  - 5.3.3 Circles
  - 5.3.4 Planes
  - 5.3.5 Conics
  - 5.3.6 Polar Coordinates
  - 5.3.7 Parametric Curves
  - 5.3.8 Surfaces
  - 5.3.9 Curvilinear Coordinates
  - 5.3.10 Distance Formula
- 5.4 Projective Geometry
- 5.5 Differential Geometry
- 5.6 Algebraic Geometry
- 5.7 Topology
  - 5.7.1 Point Set Topology
  - 5.7.2 General Topology
  - 5.7.3 Differential Topology
  - 5.7.4 Algebraic Topology
- 5.8 Trigonometry
  - 5.8.1 Angles
  - 5.8.2 Trigonometric Functions
  - 5.8.3 Inverse Trigonometric Functions
  - 5.8.4 Trigonometric Identities
  - 5.8.5 Trigonometric Equations
  - 5.8.6 Roots of Unity
  - 5.8.7 Spherical Trigonometry
- 5.9 Fractal Geometry

6.0 Calculus

- 6.1 Single Variable
  - 6.1.1 Functions
  - 6.1.2 Limits
  - 6.1.3 Continuity
  - 6.1.4 Differentiation
  - 6.1.5 Integration
  - 6.1.6 Series
- 6.2 Several Variables
  - 6.2.1 Functions of Several Variables
  - 6.2.2 Limits
  - 6.2.3 Continuity

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- 6.2.4 Partial Derivatives
- 6.2.5 Multiple integrals
- 6.2.6 Taylor Series
- 6.3 Advanced Calculus
  - 6.3.1 Vector Valued Functions
  - 6.3.2 Line Integrals
  - 6.3.3 Surface Integrals
  - 6.3.4 Stokes Theorem
  - 6.3.5 Linear spaces
  - 6.3.6 Fourier Series
  - 6.3.7 Orthogonal Functions
- 6.4 Tensor Calculus
- 6.5 Calculus of Variations
- 6.6 Operational Calculus

7.0 Analysis

- 7.1 Real Analysis
  - 7.1.1 Metric Spaces
  - 7.1.2 Convergence
  - 7.1.3 Continuity
  - 7.1.4 Differentiation
  - 7.1.5 Integration
  - 7.1.6 Measure Theory
- 7.2 Complex Analysis
  - 7.2.1 Convergence
  - 7.2.2 Infinite Series
  - 7.2.3 Analytic Functions
  - 7.2.4 Integration
  - 7.2.5 Contour Integrals
  - 7.2.6 Conformal Mappings
  - 7.2.7 Several Complex Variables
- 7.3 Numerical Analysis
  - 7.3.1 Computer Arithmetic
  - 7.3.2 Solutions of Equations
  - 7.3.3 Solutions of Systems
  - 7.3.4 Interpolation
  - 7.3.5 Numerical Differentiation
  - 7.3.6 Numerical Integration
  - 7.3.7 Numerical Solutions of ODEs
  - 7.3.8 Numerical Solutions of PDEs
- 7.4 Signal Analysis
  - 7.4.1 Fourier Series
- 7.5 Fourier Transforms
  - 7.5.1 Filters
  - 7.5.2 Noise
  - 7.5.3 Sampling Theory
  - 7.5.4 Wavelet Analysis
  - 7.5.5 Data Compression
  - 7.5.6 Image Processing
- 7.6 Functional Analysis
  - 7.6.1 Hilbert Spaces
  - 7.6.2 Banach Spaces
  - 7.6.3 Topological Spaces
  - 7.6.4 Locally Convex Spaces
  - 7.6.5 Bounded Operators
  - 7.6.6 Spectral Theorem
  - 7.6.7 Unbounded Operators
- 7.7 Harmonic Analysis
- 7.8 Global Analysis

8.0 Differential and Difference Equations

- 8.1 Ordinary Differential Equations
  - 8.1.1 First Order
  - 8.1.2 Second Order
  - 8.1.3 Linear Oscillations
  - 8.1.4 Nonlinear Oscillations
  - 8.1.5 Systems of Differential Equations
  - 8.1.6 Sturm - Liouville Problems
  - 8.1.7 Special Functions
  - 8.1.8 Power Series Methods
  - 8.1.9 Laplace Transforms
- 8.2 Partial Differential Equations
  - 8.2.1 First Order
  - 8.2.2 Elliptic
  - 8.2.3 Parabolic
  - 8.2.4 Hyperbolic
  - 8.2.5 Integral Transforms
  - 8.2.6 Integral Equations
  - 8.2.7 Potential Theory
  - 8.2.8 Nonlinear Equations
  - 8.2.9 Symmetries and Integrability
- 8.3 Difference Equations
  - 8.3.1 First Order
  - 8.3.2 Second Order
  - 8.3.3 Linear Systems
  - 8.3.4 Z-Transforms
  - 8.3.5 Orthogonal Polynomials
- 8.4 Dynamical Systems
  - 8.4.1 1D Maps
  - 8.4.2 2D Maps
  - 8.4.3 Lyapunov Exponents
  - 8.4.4 Bifurcations
  - 8.4.5 Fractals
  - 8.4.6 Differentiable Dynamics
  - 8.4.7 Conservative Dynamics
  - 8.4.8 Chaos
  - 8.4.9 Complex Dynamical Systems

9.0 Statistics and Probability

- 9.1 Statistics
  - 9.1.1 Sampling
  - 9.1.2 Expectation Value and Variance
  - 9.1.3 Linear Regression
  - 9.1.4 Nonlinear Regression
  - 9.1.5 Queuing Theory
  - 9.1.6 Bayesian Statistics
- 9.2 Probability
  - 9.2.1 Brownian Motion
  - 9.2.2 Random Variables
    - 9.2.2.1 Discrete Distributions
    - 9.2.2.2 Continuous

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- Distributions
  - 9.2.2.3 Expectation Value
- 9.2.3 Central Limit Theorem
- 9.2.4 Markov Chains
- 9.2.5 Probability Measures
- 9.2.6 Stochastic Processes

9.3 Data

- 9.3.1 Data Representation
  - 9.3.1.1 Bar graph
  - 9.3.1.2 Box-and-whiskers plot
  - 9.3.1.3 Circle graph/pie graph
  - 9.3.1.4 Graphing
  - 9.3.1.5 Histogram
  - 9.3.1.6 Line-of-best-fit
  - 9.3.1.7 Line plot
  - 9.3.1.8 Pictograph
  - 9.3.1.9 Scatter plot
  - 9.3.1.10 Stem-and-leaf plot
  - 9.3.1.11 Table
- 9.3.2 Data Collection
  - 9.3.2.1 Experiment
  - 9.3.2.2 Hypothesis
  - 9.3.2.3 Sampling
  - 9.3.2.4 Survey
- 9.3.3 Data Analysis
  - 9.3.3.1 Measures of Central Tendency
    - 9.3.3.1.1 Mean
    - 9.3.3.1.2 Median
    - 9.3.3.1.3 Mode
  - 9.3.3.2 Correlation
  - 9.3.3.3 Distribution

10.0 Applied Mathematics

- 10.1 Mathematical Physics
- 10.2 Mathematical Economics
- 10.3 Mathematical Biology
- 10.4 Mathematics for Business
- 10.5 Engineering Mathematics
- 10.6 Mathematical Sociology
- 10.7 Mathematics for Social Sciences
- 10.8 Mathematics for Computer Science

11.0 Mathematics History

- 11.1 General
- 11.2 Famous Problems
- 11.3 Biographies of Mathematicians

C. Individual Comments

The following is a synopsis of comments made to the February 2002 core taxonomy. They are grouped into comments on procedure and comments on specifics in the taxonomy. There may be some comments on procedure in the latter to keep comments relatively in tact.

Comments on Procedure

Robby Robson

> Just glancing through this, my reaction is that there is merit to the critiques, but the question being asked is not whether the taxonomy is perfect but whether it is workable. If the small committee responsible for this wants to make some tweaks, that's fine, and any truly serious problems should be fixed, but I rather think we are at the point where making one person happy will make someone else unhappy. If we want to reach consensus within the community, the best way is to release the taxonomy.
>
> COMMENT: In doing so, people will jump in and make comments without having the perspective gained from wrestling with a problem for several years. This is part of the process, and it will help all involved to have good documentation. For example, the comments made below about pi and e may be anatomically correct, but the decision to call out pi and e was made on the basis of search patterns observed at the Math Forum and elsewhere. It must be recognized that the importance and indeed containment relationship among subject areas can seldom be uniformly defined across all targeted user communities. If there is real uptake, it will be necessary to explain what points of view were considered and why certain decisions were reached, and to have a good process for review and revision.
>
> I know we went over this in Washington. My point is that we need to shift our thinking away from production and towards dissemination, revision, and maintenance. I am still recommending we go with what we have and get on with next part of the job.

Russ Herman, iLumina

> The majority of the proposed taxonomy has been out for close to a year. The suggestions made at the last meeting are minor modifications. The committee will look at the suggestions and see if there is a need to make more modifications. Modifications will only be made, as the committee decided, if people cannot "live with the taxonomy" as it stands. There have to be good reasons to make changes. This means that people cannot "map" their data to the taxonomy as it stands. There were rules that led to the current form. As was pointed out, there may be some typos that arose from Brandon's translation,
> and these are easily fixed.

Gabriel Lugo, iLumina

> You can track the history of the taxonomy at the site http://mathforum.org/mathnsdl/
>
> At the first meeting of the math NDSL meeting in Washington on March 15 2001, we had a long discussion on the differences in taxonomies created by the Math MetaData Task force, MERLOT, iLumina, ENC, and other digital libraries represented at the meeting.
>
> We agreed that we would circulate the taxonomy amongst the digital libraries to see if we could reach a consensus.

Comments on Taxonomy Items

Gene Klotz and MathForum (Lee, Jay and Richard)

- pi and e may, indeed, be "numbers," but lumping them on the same level as natural, integer, rational, and other types of numbers condemns apples to the same display case as all species from the plant kingdom.
- Re-duplicating addition, subtraction, multiplication, and division for each of the arithmetic categories of "operations" and "fractions" and "decimals" seems arbitrary, redundant, and bound to cause confusion during browsing.
- "Polygons" contains rectangles, squares, and trapezoids. Why not begin just with quadrilaterals? Will I find no squares under regular polygons?
- There is one point where the taxonomy is 5 levels deep, under measures of central tendency. Is this necessary? It may cause presentation problems and readability problems. Also, that section isn't numbered correctly.
- 1.2.1.5 Square Roots and 1.1.6 Exponents (which should be 1.2.6) ...how do those relate to each other? And to 1.2.6.3 Powers.
- Many of the geometrical items in the Elementary Math section bother me, perhaps because they're more obviously Geometry (unlike, say, basic operations, which seems pretty clearly Elementary Math to me). They get repeated or ignored in the Geometry categories. This is an artificial use of the idea "topic" to mean "level".
- 3.4.6 Famous Problems crosses a number of categories. Or rather, it should. Does that mean it should maybe be a resource type, or some other sort of descriptor?
- Oh, now, I'm pretty familiar with 5.3.10 Distance Formula, but I've no clue what 5.3.9 Curvilinear Coordinates are. Perhaps Distance Formula might be associated with 5.1.1 Measurement or somewhere around 5.2.1 Lines and Planes?
- Fractal Geometry is Geometry and not Fractals?
- Don't [doesn't the] Fourier Series (7.4.1) get used in a whole lot more places than just 7.4 Signal Analysis?
- 7.5 Fourier Transforms seem like a whole nother type of category from the other categories at its level: Real Analysis, Complex Analysis, Numerical An., Signal An., Functional and Harmonic and Global Analysis. Jay agrees this time, and has further suspicions to which I cannot do justice, since I don't understand them. (He wouldn't call them suspicions, though.)
- Brownian Motion says physics to me, not math.
- A note on Lee's note: you wonder about the necessity for five levels of depth, and the possibility of that being confusing. I wonder about the "Arbitrary levels of detail" allowed below the visible structure in later implementations [viz secondary goals]. I also wonder about the lack of depth in other areas. [If so, you need more teachers *of that level* involved in making it up. Y'all know >too much, so your taxonomy becomes impenetrable to the average plebe.] I actually think there's a second level task which involves mapping structures/vocabulary that the end user recognizes, implemented both as browsing and an intelligent search or query process that walks the users closer to his/her chosen topic.
- I agreed with much the Richard said but I think I disagree about the repetition of operations under topics such as Fractions. I think a given catalog has to be able to transport users, without them necessarily knowing it from one to the other. If I am pursuing adding fractions, I should be able to see Fractions before I see Addition, just as they have it, but if I choose to go down through Operations or happen to see Addition at the lower level, then there should be a gating process that offers them a choice about what they're adding and takes them to the right place in the rest of the hierarchy.
- Similarly, students will have to be able to search on factoring and be presented with choices about numbers or polynomials, for instance.
- Richard also mentioned 'e' and 'pi'. These could be grouped under transcendental. Should this be presented as a subset of irrational?
- In a related issue [to geometric concepts], the topics under Patterns and Sequences are not repeated places where they might be, for instance the elementary topics of tiling and tessellation, or symmetry, have much more sophisticated versions that a teacher would expect to see under Geometry, wouldn't they?
- Yes, there is also Famous Problems under the Math History section, which seems more appropriate. Why single it out under Number Theory?
- Some of the topic trees seem less expertly developed than others. Is this really the structure of the Probability taxonomy. And what happens for more early treatments, focusing on concepts such as Combinations and Permutations and Choosing?

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- Certainly the depth of trees seems somewhat arbitrary, for instance the shallowness of the Discrete Mathematics topics.[Note from Gene: In olden days I had a colleague who referred to Discrete Mathematics as After Dinner Mathematics. Perhaps the tree properly reflects the subject? :-)]

David Barnes and NCTM

Elementary Mathematics – This seems artificial. Algebra, geometry, number all fall inside and outside. And Probability and Data are not included, but are part of the mathematics curricula from K-12.

Change Elementary Mathematics to Numbers and Computation
Change Numbers to Number Concepts and Definitions
- Add Golden Ratio
- Add Fractions, decimal, and percents
- Add Prime Numbers
- Add Comparison of Numbers
- Move Pi and e under irrational or remove.

Remove Arithmetic and move areas under Numbers and Computation
Change Operations to Computation Concepts
- Change Square Roots to Roots or Square Roots and Other Roots.
- Add Divisibility and Factorization
- Add Exponents.

Add Whole Number and Integer Computation w/ sublist same as Operations.
Change Fractions to be Computation with Fractions (and so on for the other operations)
Start all sub lists with Addition, Subtraction, Multiplication, and Division for consistency, and place other items at the end.
Estimation seems too broad where it is. Add to each Computation subgroup. This doesn't fix the problem (just makes it more localized – estimation on fractions?)
Change Patterns and Sequences to Patterns, Relationships, and Sequences and move under Numbers and Computation
- Move Geometric Patterns and Tilings and Tessellations to Geometry

Under Estimation
- Add Quantity
- Add Operation
- Add Measurement
- Add Contextual/Problem Solving

Under Exponent
- Add Integer
- Add Fractional
- Add Negative
- Add Irrational

Under Discrete Mathematics
- Add Recursion

Change Geometry 5.0 to Geometry and Topology
Add lead subcategory Geometry 5.1 with the following categories underneath it. (As was done with Algebra and Number Theory)
- Object Recognition and Characteristics
  - Plane Figures
    - (Include Plane figures)
  - Solid Figures
    - (include 3D figures)
- Congruence
- Similarity
- Transformation
  - Slide or Translations
  - Reflection
  - Rotation

- Glide Reflection
- Symmetry
- Coordinate Geometry
- Spatial Sense
- Tilings and Tessellations
- Geometric Patterns
- Add Other Quadrilaterals or subsume current ones under the heading Quadrilaterals.
- Under Triangles
  - Add Congruence
  - Add Similarity
  - Add Centers
- Under Polygons add Irregular Polygons
- Under Circles
  - Add Tangents
  - Properties of Circles
  - Parts of Circles

Add measurement as a top level concept.

Measurement

(add) Measurement Concepts
- Units of Measure
  - Standard Units
  - Metric System
  - Non-standard Units
- Length/distance
- Area
- Surface Area
- Volume
- Weight and Mass
- Temperature
- Time
- Speed
- Money

Measurement Applications
- Linear
  - Distance
  - Circumference
  - Perimeter
- Area
  - Triangles
  - Rectangles
  - Other Polygons
  - Circles
  - Non-standard shapes
- Surface Area
- Scale

Under Algebra and Number Theory

Consider separating Algebra and Number Theory.

Combine Functions and Equations into one category.

Graphing seems naked. Either needs list of function types underneath or removed.

Systems of Linear Equations should be moved or also included under Algebra.

Rename Statistics and Probability to Data, Statistics and Probability
- Rename Statistics to Statistics and Data Analysis
  - Move categories under Data Analysis under this heading.
  - add Inference and Prediction
  - Add Standard Deviation
  - Add Confidence Interval

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Move Hypothesis to Data Analysis
Under Probability add:
Theoretical Probabilities
Sample Space
Single Events
Compound Events
Independent and Dependent
Expected Value

Gabriel Lugo, iLumina

- Some very minor typographical changes to the Taxonomy.
    1. Elementary Mathematics should be 1.1 and not 1.0
    2. The automatic tree formatting is a bit messed up. For example, 1.3Patterns and sequences and subsequent branches have a spacing problem and are in the wrong font.
    3. In the word document Some numbers in the subtrees are light gray and some are bold face. Same problem with the auto-formatting.
    4. Would like to suggest that Curvilinear Coordinates in section 5.3 Analytic Geometry be moved to 6.3.5 Advanced Calculus. This makes more sense and also reduces the number of branches of 5.3 to nine.
- Yes, I originally put curvilinear coordinates in the analytic geometry section precisely for the reasons that you stated.
- But the real value of treating this topic is the context of looking at Gradients, Curls, divergence and Laplacians which clearly falls in the arena of advanced calculus.
- Originally, the taxonomy had two branches:
    1. Probability, 2. Statistics.

  The MathForum group combined it into one branch:
    1. Probability / Statistics / Measurement / Data

  The iLumina group went back to two, but modified the sub-branches
    1. Probability, 2. Statistics.

  The third branch on Data was reintroduced at the last meeting in Washington.
- I think the last branch on Data is already included in the first two. Unfortunately I could not be at that meeting so I was not privy to the discussions on this change.

Susan Kornstein, College Board (AP Math and Statistics)

- I have some questions about the taxonomy for Statistics and Probability 9.0. I do not know the history behind some of the choices, but some are confusing to me.
- Do we really need this number of branches for statistics? It would be easier to map to this list if there were fewer. Calculus is a better model for how to do this.
- Sampling appears both under Statistics and also under Data Collection. I think Data Representation should include numerical (measures of center and measures of spread) as well as graphical summaries. I am not sure where the general rules for probability (a strand in the high school curriculum would go on this list).
- The AP Statistics course description divides into 4 units;
  1. Exploring Data (graphical displays, numerical summaries, regression)
  2. Planning a Study (data collection, surveys, design of experiments)
  3. Probability and simulation (probability rules, random variables, sampling distributions)
  4. Statistical Inference (confidence intervals, tests of significance)

Frank Wattenberg, MathDL

A quick comment from someone coming in very late to the conversation and whose comments should therefore be taken with a bucket of salt-- probability, statistics, and data are all different subjects. It makes sense to me to have three distinct (sub)branches.

D. NCTM Numbers and Computation Proposal

- 1.1 Numbers and Computation
- 1.2 Number Concepts and Definitions
  - 1.2.1 Natural
  - 1.2.2 Integers
  - 1.2.3 Prime Numbers
  - 1.2.4 Fractions Decimals and Percents
  - 1.2.5 Rational
  - 1.2.6 Irrational
    - 1.2.6.1 pi
    - 1.2.6.2 e
    - 1.2.6.3 Golden Ratio
  - 1.2.7 Algebraic
  - 1.2.8 Real
  - 1.2.9 Complex
  - 1.2.10 Comparison of Numbers
- 1.3 Computation Concepts
  - 1.3.1 Addition
  - 1.3.2 Subtraction
  - 1.3.3 Multiplication
  - 1.3.4 Division
  - 1.3.5 Exponents
  - 1.3.6 Roots
  - 1.3.7 Factorials
  - 1.3.8 Divisibility and Factorization
- 1.4 Computation with Whole Numbers
  - 1.4.1 Addition
  - 1.4.2 Subtraction
  - 1.4.3 Multiplication
  - 1.4.4 Division
  - 1.4.5 Estimation
- 1.5 Computation with Integers
  - 1.5.1 Addition
  - 1.5.2 Subtraction
  - 1.5.3 Multiplication
  - 1.5.4 Division
  - 1.5.5 Estimation
- 1.6 Computation with Fractions
  - 1.6.1 Addition
  - 1.6.2 Subtraction
  - 1.6.3 Multiplication
  - 1.6.4 Division
  - 1.6.5 Estimation
  - 1.6.6 Ratio and Proportion
  - 1.6.7 Equivalent Fractions
- 1.7 Computation with Decimals
  - 1.7.1 Addition
  - 1.7.2 Subtraction
  - 1.7.3 Multiplication
  - 1.7.4 Division
  - 1.7.5 Estimation
  - 1.7.6 Percents
- 1.8 Computation with Exponents
  - 1.8.1 Multiplication
  - 1.8.2 Division
  - 1.8.3 Powers
  - 1.8.4 Estimation
  - 1.8.5 Integer
  - 1.8.6 Fractional
  - 1.8.7 Negative
  - 1.8.8 Irrational
- 1.9 Estimation
  - 1.9.1 Quantity
  - 1.9.2 Operation
  - 1.9.3 Measurement
  - 1.9.4 Contextual Problem Solving
- 1.10 Patterns Relationships and Sequences
  - 1.10.1 Golden Ratio
  - 1.10.2 Fibonacci Sequence
  - 1.10.3 Arithmetic Sequence
  - 1.10.4 Geometric Sequence
- 1.11 Shapes and Figures
  - 1.11.1 Plane shapes
  - 1.11.2 Spatial Sense
  - 1.11.3 Symmetry
  - 1.11.4 Similar Figures
  - 1.11.5 Solid Shapes

- 2.0 Measurement
- 2.1 Measurement Concepts
  - 2.1.1 Units of Measure
    - 2.1.1.1 Standard Units
    - 2.1.1.2 Metric System
    - 2.1.1.3 Nonstandard Units
  - 2.1.2 Length and Distance
  - 2.1.3 Area
  - 2.1.4 Surface Area
  - 2.1.5 Volume
  - 2.1.6 Weight and Mass
  - 2.1.7 Temperature
  - 2.1.8 Time
  - 2.1.9 Speed
  - 2.1.10 Money
- 2.2 Measurment Applications
  - 2.2.1 Linear
    - 2.2.1.1 Distance
    - 2.2.1.2 Circumference
    - 2.2.1.3 Perimeter
  - 2.2.2 Area
    - 2.2.2.1 Triangles
    - 2.2.2.2 Rectangles
    - 2.2.2.3 Other Polygons
    - 2.2.2.4 Circles
    - 2.2.2.5 Nonstandard shapes
  - 2.2.3 Surface Area
- 2.2.4 Scale

E. Specific Changes Made by the Committee

1. Change Elementary Mathematics to Numbers and Computation.
2. Change Numbers to Number Concepts.
3. Add to Number Concepts: Famous Numbers.
4. Move Pi and e under Famous Numbers.
5. Add 0, i, Golden Mean under Famous Numbers.
6. Change Square Roots to Roots.
7. Under Exponents add Rational, Negative.
8. Add to Arithmetic/Operations: Factoring, Properties of Operations.
9. Order common operations the same in all subtopics (Addition, Subtraction, ...).
10. Move Estimation Under Operations.
11. Move Geometric Patterns, Tilings and Tesselations, and Golden Ratio under Geometry/Patterns.
12. Add to Patterns and Sequences: Number Patterns.
13. Delete Shapes and Figures.
14. Add to Units of Measurement: Non-standard Units.
15. Move Scale to Higher level under Measurement.
16. Add to Measurement: Weight and Mass, Temperature, Time, Speed, Money.
17. Add to Area Nonstandard Shapes.
18. Delete Induction.
19. Add to Algebra: Algebraic Manipulation.
20. Under Algebra: Change Graphing to Graphing Techniques.
21. Add to Discrete Mathematics/Combinatorics: Combinations, Permutations.
22. Add to Discrete Mathematics: Recursion.
23. Change Geometry to Geometry and Topology.
24. Move Geometric Proof one level up.
25. Rename Parallel Lines and Perpendicular Lines: Lines and Planes.
26. Under Triangles rename Properties of Right Triangles to Properties.
27. Move Congruence, Similarity into Triangles. (They may have originally been there.)
28. Delete all topics under Polygons except Regular.
29. Add to Polygons: Properties, Congruence, Similarity and Irregular.
30. Under Solid Geometry delete Lines and Planes.
31. Rename Angles in Solid Geometry: Dihedral Angles.
32. Under Plane Geometry Add Transformations.
33. Under Transformations Add Translation, Rotation, Reflection, Scaling.
34. Add under Geometry: Symmetry.
35. Under Analytic Geometry move Curvilinear Coordinates after Stoke's Theorem under Advanced Calculus.
36. Delete Signal Analysis and Fourier Transforms and subtopics. (Originally Fourier Transforms was under Signal Analysis.)
37. Add to Analysis: Integral Transforms, Signal Analysis.
38. Add to Integral Transforms: Fourier Transforms, Laplace, Hankel Transforms, Wavelets, Other Transforms.
39. Add to Signal Analysis: Sampling Theory, Filters, Noise, Data Compression, Image Processing.
40. Delete Expectation Value and Variance under Statistics.
41. Add Variance to Random Variables.
42. Rename Central Limit Theorem to Limit Theorems and add subtopics Central Limit Theorem, Laws of Large Numbers.
43. Add Elementary Probability to Probability with subtopics: Sample Space, Events, Independence, Combinations and Permutations.
44. Add to Statistics: Confidence Testing.
45. Under Data Representation, collapse topics to Graphs, BoxPlots, StemPlots, Tables.
46. Rename Experiment as Experimental Design.
47. Remove Hypothesis.
48. Delete Measures of Central Tendency and move Mean, Median and Mode to a higher level.
49. Add Standard Deviation after Mode.
50. Move Data Analysis under Statistics and rename it Elementary Statistics.
51. Change Computatability, Decidability and Recursion in 2.0 to Computatability and Decidability.
52. Add to Applied Mathematics: Mathematics for Humanities, Consumer Mathematics.

F. The Final Taxonomy

- 1.0 Numbers and Computation
  - 1.1 Number Concepts
    - 1.1.1 Natural
    - 1.1.2 Integers
    - 1.1.3 Rational
    - 1.1.4 Irrational
    - 1.1.5 Algebraic
    - 1.1.6 Real
    - 1.1.7 Complex
    - 1.1.8 Famous Numbers
      - 1.1.8.1 0
      - 1.1.8.2 pi
      - 1.1.8.3 e
      - 1.1.8.4 i
      - 1.1.8.5 Golden Mean
  - 1.2 Arithmetic
    - 1.2.1 Operations
      - 1.2.1.1 Addition
      - 1.2.1.2 Subtraction
      - 1.2.1.3 Multiplication
      - 1.2.1.4 Division
      - 1.2.1.5 Roots
      - 1.2.1.6 Factorials
      - 1.2.1.7 Factoring
      - 1.2.1.8 Properties of Operations
      - 1.2.1.9 Estimation
    - 1.2.2 Fractions
      - 1.2.2.1 Addition
      - 1.2.2.2 Subtraction
      - 1.2.2.3 Multiplication
      - 1.2.2.4 Division
      - 1.2.2.5 Ratio and Proportion
      - 1.2.2.6 Equivalent Fractions
    - 1.2.3 Decimals
      - 1.2.3.1 Addition
      - 1.2.3.2 Subtraction
      - 1.2.3.3 Multiplication
      - 1.2.3.4 Division
      - 1.2.3.5 Percents
    - 1.2.4 Comparison of numbers
    - 1.2.5 Exponents
      - 1.2.5.1 Multiplication
      - 1.2.5.2 Division
      - 1.2.5.3 Powers
      - 1.2.5.4 Integer Exponents
      - 1.2.5.5 Rational Exponents
  - 1.3 Patterns and Sequences
    - 1.3.1 Number Patterns
    - 1.3.2 Fibonacci Sequence
    - 1.3.3 Arithmetic Sequence
    - 1.3.4 Geometric Sequence
  - 1.4 Measurement
    - 1.4.1 Units of Measurement
      - 1.4.1.1 Metric System
      - 1.4.1.2 Standard Units
      - 1.4.1.3 Nonstandard Units
    - 1.4.2 Linear Measure
      - 1.4.2.1 Distance
      - 1.4.2.2 Circumference
      - 1.4.2.3 Perimeter
    - 1.4.3 Area
      - 1.4.3.1 Area of Polygons
      - 1.4.3.2 Area of Circles
      - 1.4.3.3 Surface Area
      - 1.4.3.4 Nonstandard Shapes
    - 1.4.4 Volume
    - 1.4.5 Weight and Mass
    - 1.4.6 Temperature
    - 1.4.7 Time
    - 1.4.8 Speed
    - 1.4.9 Money
    - 1.4.10 Scale
- 2.0 Logic and Foundations
  - 2.1 Logic
    - 2.1.1 Venn Diagrams
    - 2.1.2 Propositional and Predicate Logic
    - 2.1.3 Methods of Proof
  - 2.2 Set Theory
    - 2.2.1 Sets and Set Operations
    - 2.2.2 Relations and Functions
    - 2.2.3 Cardinality
    - 2.2.4 Axiom of Choice
  - 2.3 Computability and Decidability
  - 2.4 Model Theory
- 3.0 Algebra and Number Theory
  - 3.1 Algebra
    - 3.1.1 Graphing Techniques
    - 3.1.2 Algebraic Manipulation
    - 3.1.3 Functions
      - 3.1.3.1 Linear
      - 3.1.3.2 Quadratic
      - 3.1.3.3 Polynomial
      - 3.1.3.4 Rational
      - 3.1.3.5 Exponential
      - 3.1.3.6 Logarithmic
      - 3.1.3.7 Piece-wise
      - 3.1.3.8 Step
    - 3.1.4 Equations
      - 3.1.4.1 Linear
      - 3.1.4.2 Quadratic
      - 3.1.4.3 Polynomial

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- 3.1.4.4 Rational
- 3.1.4.5 Exponential
- 3.1.4.6 Logarithmic
- 3.1.4.7 Systems
- 3.1.5 Inequalities
- 3.1.6 Matrices
- 3.1.7 Sequences and Series
- 3.1.8 Algebraic Proof
- 3.2 Linear Algebra
  - 3.2.1 Systems of Linear Equations
  - 3.2.2 Matrix algebra
  - 3.2.3 Vectors in R3
  - 3.2.4 Vector Spaces
  - 3.2.5 Linear Transformations
  - 3.2.6 Eigenvalues and Eigenvectors
  - 3.2.7 Inner Product Spaces
- 3.3 Abstract Algebra
  - 3.3.1 Groups
  - 3.3.2 Rings and Ideals
  - 3.3.3 Fields
  - 3.3.4 Galois Theory
  - 3.3.5 Multilinear Algebra
- 3.4 Number Theory
  - 3.4.1 Integers
  - 3.4.2 Primes
    - 3.4.2.1 Divisibility
    - 3.4.2.2 Factorization
    - 3.4.2.3 Distributions of Primes
  - 3.4.3 Congruences
  - 3.4.4 Diophantine Equations
  - 3.4.5 Irrational Numbers
  - 3.4.6 Famous Problems
  - 3.4.7 Coding Theory
  - 3.4.8 Cryptography
- 3.5 Category Theory
- 3.6 K-Theory
- 3.7 Homological Algebra
- 3.8 Modular Arithmetic

4.0 Discrete Mathematics

- 4.1 Cellular Automata
- 4.2 Combinatorics
  - 4.2.1 Combinations
  - 4.2.2 Permutations
- 4.3 Game Theory
- 4.4 Algorithms
- 4.5 Recursion
- 4.6 Graph Theory
- 4.7 Linear Programming
- 4.8 Order and Lattices
- 4.9 Theory of Computation
- 4.10 Chaos

5.0 Geometry and Topology

- 5.1 Geometric Proof
- 5.2 Plane Geometry
  - 5.2.1 Measurement
  - 5.2.2 Lines and Planes
  - 5.2.3 Angles
  - 5.2.4 Triangles
    - 5.2.4.1 Properties
    - 5.2.4.2 Congruence
    - 5.2.4.3 Similarity
    - 5.2.4.4 Pythagorean Theorem
  - 5.2.5 Polygons
    - 5.2.5.1 Properties
    - 5.2.5.2 Regular
    - 5.2.5.3 Irregular
    - 5.2.5.4 Congruence
    - 5.2.5.5 Similarity
  - 5.2.6 Circles
  - 5.2.7 Patterns
    - 5.2.7.1 Geometric Patterns
    - 5.2.7.2 Tilings and Tessellations
    - 5.2.7.3 Symmetry
    - 5.2.7.4 Golden Ratio
  - 5.2.8 Transformations
    - 5.2.8.1 Translation
    - 5.2.8.2 Rotation
    - 5.2.8.3 Reflection
    - 5.2.8.4 Scaling
- 5.3 Solid Geometry
  - 5.3.1 Dihedral Angles
  - 5.3.2 Spheres
  - 5.3.3 Cones
  - 5.3.4 Cylinders
  - 5.3.5 Pyramids
  - 5.3.6 Prisms
  - 5.3.7 Polyhedra
- 5.4 Analytic Geometry
  - 5.4.1 Cartesian Coordinates
  - 5.4.2 Lines
  - 5.4.3 Circles
  - 5.4.4 Planes
  - 5.4.5 Conics
  - 5.4.6 Polar Coordinates
  - 5.4.7 Parametric Curves
  - 5.4.8 Surfaces
  - 5.4.9 Distance Formula
- 5.5 Projective Geometry
- 5.6 Differential Geometry
- 5.7 Algebraic Geometry
- 5.8 Topology
  - 5.8.1 Point Set Topology
  - 5.8.2 General Topology
  - 5.8.3 Differential Topology
  - 5.8.4 Algebraic Topology
- 5.9 Trigonometry
  - 5.9.1 Angles
  - 5.9.2 Trigonometric Functions
  - 5.9.3 Inverse Trigonometric Functions
  - 5.9.4 Trigonometric

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- Identities
- 5.9.5 Trigonometric Equations
- 5.9.6 Roots of Unity
- 5.9.7 Spherical Trigonometry
- 5.10 Fractal Geometry

6.0 Calculus

- 6.1 Single Variable
  - 6.1.1 Functions
  - 6.1.2 Limits
  - 6.1.3 Continuity
  - 6.1.4 Differentiation
  - 6.1.5 Integration
  - 6.1.6 Series
- 6.2 Several Variables
  - 6.2.1 Functions of Several Variables
  - 6.2.2 Limits
  - 6.2.3 Continuity
  - 6.2.4 Partial Derivatives
  - 6.2.5 Multiple integrals
  - 6.2.6 Taylor Series
- 6.3 Advanced Calculus
  - 6.3.1 Vector Valued Functions
  - 6.3.2 Line Integrals
  - 6.3.3 Surface Integrals
  - 6.3.4 Stokes Theorem
  - 6.3.5 Curvilinear Coordinates
  - 6.3.6 Linear spaces
  - 6.3.7 Fourier Series
  - 6.3.8 Orthogonal Functions
- 6.4 Tensor Calculus
- 6.5 Calculus of Variations
- 6.6 Operational Calculus

7.0 Analysis

- 7.1 Real Analysis
  - 7.1.1 Metric Spaces
  - 7.1.2 Convergence
  - 7.1.3 Continuity
  - 7.1.4 Differentiation
  - 7.1.5 Integration
  - 7.1.6 Measure Theory
- 7.2 Complex Analysis
  - 7.2.1 Convergence
  - 7.2.2 Infinite Series
  - 7.2.3 Analytic Functions
  - 7.2.4 Integration
  - 7.2.5 Contour Integrals
  - 7.2.6 Conformal Mappings
  - 7.2.7 Several Complex Variables
- 7.3 Numerical Analysis
  - 7.3.1 Computer Arithmetic
  - 7.3.2 Solutions of Equations
  - 7.3.3 Solutions of Systems
  - 7.3.4 Interpolation
  - 7.3.5 Numerical Differentiation
  - 7.3.6 Numerical Integration
  - 7.3.7 Numerical Solutions of ODEs
  - 7.3.8 Numerical Solutions of PDEs
- 7.4 Integral Transforms
  - 7.4.1 Fourier Transforms
  - 7.4.2 Laplace Transforms
  - 7.4.3 Hankel Transforms
  - 7.4.4 Wavelets
  - 7.4.5 Other Transforms
- 7.5 Signal Analysis
  - 7.5.1 Sampling Theory
  - 7.5.2 Filters
  - 7.5.3 Noise
  - 7.5.4 Data Compression
  - 7.5.5 Image Processing
- 7.6 Functional Analysis
  - 7.6.1 Hilbert Spaces
  - 7.6.2 Banach Spaces
  - 7.6.3 Topological Spaces
  - 7.6.4 Locally Convex Spaces
  - 7.6.5 Bounded Operators
  - 7.6.6 Spectral Theorem
  - 7.6.7 Unbounded Operators
- 7.7 Harmonic Analysis
- 7.8 Global Analysis

8.0 Differential and Difference Equations

- 8.1 Ordinary Differential Equations
  - 8.1.1 First Order
  - 8.1.2 Second Order
  - 8.1.3 Linear Oscillations
  - 8.1.4 Nonlinear Oscillations
  - 8.1.5 Systems of Differential Equations
  - 8.1.6 Sturm Liouville Problems
  - 8.1.7 Special Functions
  - 8.1.8 Power Series Methods
  - 8.1.9 Laplace Transforms
- 8.2 Partial Differential Equations
  - 8.2.1 First Order
  - 8.2.2 Elliptic
  - 8.2.3 Parabolic
  - 8.2.4 Hyperbolic
  - 8.2.5 Integral Transforms
  - 8.2.6 Integral Equations
  - 8.2.7 Potential Theory
  - 8.2.8 Nonlinear Equations
  - 8.2.9 Symmetries and Integrability
- 8.3 Difference Equations
  - 8.3.1 First Order
  - 8.3.2 Second Order
  - 8.3.3 Linear Systems
  - 8.3.4 Z Transforms
  - 8.3.5 Orthogonal

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- Polynomials
- 8.4 Dynamical Systems
  - 8.4.1 1D Maps
  - 8.4.2 2D Maps
  - 8.4.3 Lyapunov Exponents
  - 8.4.4 Bifurcations
  - 8.4.5 Fractals
  - 8.4.6 Differentiable Dynamics
  - 8.4.7 Conservative Dynamics
  - 8.4.8 Chaos
  - 8.4.9 Complex Dynamical Systems
- 9.0 Statistics and Probability
  - 9.1 Data
    - 9.1.1 Data Collection
      - 9.1.1.1 Experimental Design
      - 9.1.1.2 Sampling
      - 9.1.1.3 Survey
    - 9.1.2 Data Representation
      - 9.1.2.1 Graphs
      - 9.1.2.2 BoxPlots
      - 9.1.2.3 StemPlots
      - 9.1.2.4 Tables
  - 9.2 Statistics
    - 9.2.1 Elementary Statistics
      - 9.2.1.1 Mean
      - 9.2.1.2 Median
      - 9.2.1.3 Mode
      - 9.2.1.4 Standard Deviation
      - 9.2.1.5 Correlation
      - 9.2.1.6 Distribution
    - 9.2.2 Sampling
    - 9.2.3 Linear Regression
    - 9.2.4 Nonlinear Regression
    - 9.2.5 Queuing Theory
    - 9.2.6 Bayesian Statistics
    - 9.2.7 Confidence Testing
  - 9.3 Probability
    - 9.3.1 Elementary Probability
      - 9.3.1.1 Sample Space
      - 9.3.1.2 Events
      - 9.3.1.3 Independence
      - 9.3.1.4 Combinations and Permutations
    - 9.3.2 Random Variables
      - 9.3.2.1 Discrete Distributions
      - 9.3.2.2 Continuous Distributions
      - 9.3.2.3 Expected Value
      - 9.3.2.4 Variance
    - 9.3.3 Limit Theorems
      - 9.3.3.1 Central Limit Theorem
      - 9.3.3.2 Laws of Large Numbers
    - 9.3.4 Brownian Motion
    - 9.3.5 Markov Chains
    - 9.3.6 Probability Measures
    - 9.3.7 Stochastic Processes
- 10.0 Applied Mathematics
  - 10.1 Mathematical Physics
  - 10.2 Mathematical Economics
  - 10.3 Mathematical Biology
  - 10.4 Mathematics for Business
  - 10.5 Engineering Mathematics
  - 10.6 Mathematical Sociology
  - 10.7 Mathematics for Social Sciences
  - 10.8 Mathematics for Computer Science
  - 10.9 Mathematics for Humanities
  - 10.10 Consumer Mathematics
- 11.0 Mathematics History
  - 11.1 General
  - 11.2 Famous Problems
  - 11.3 Biographies of Mathematicians